Mathematics

Solutions for all

Learner's Book

9

Schools Development Unit

MACMILLAN
Solutions for all Mathematics Grade 9 Learner’s Book

© Schools Development Unit, 2013
© Illustrations and design Macmillan South Africa (Pty) Ltd, 2013

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, photocopying, recording, or otherwise, without the prior written permission of the copyright holder or in accordance with the provisions of the Copyright Act, 1978 (as amended).

Any person who commits any unauthorised act in relation to this publication may be liable for criminal prosecution and civil claims for damages.

First published 2013

13 15 17 16 14
0 2 4 6 8 10 9 7 5 3 1

Published by
Macmillan South Africa (Pty) Ltd
Private Bag X19
Northlands
2116
Gauteng
South Africa

Typeset by Candice Pretorius
Cover image from Gallo Images
Cover design by Deevine Design
Illustrations by MPS and Alex van Houwelingen

Photographs by:
AAI Fotostock: page 352 (top right), 467
African Media Online: page 352 (top left)
Fotostock: page 224
Gallo Images: page 384 and 385
Greatstock/Corbis: page 352 (bottom right), 385
Schools Development Unit: page 354

WIP PDF: 4522M000

It is illegal to photocopy any page of this book without written permission from the publishers.

The publishers have made every effort to trace the copyright holders. If they have inadvertently overlooked any, they will be pleased to make the necessary arrangements at the first opportunity.

The publishers would like to thank those organisations and individuals we have already approached and from whom we are anticipating permission.
## Contents

### Term 1

<table>
<thead>
<tr>
<th>Unit</th>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Whole numbers to real numbers</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Integers</td>
<td>35</td>
</tr>
<tr>
<td>3</td>
<td>Common fractions</td>
<td>47</td>
</tr>
<tr>
<td>4</td>
<td>Decimal fractions</td>
<td>62</td>
</tr>
<tr>
<td>5</td>
<td>Exponents</td>
<td>72</td>
</tr>
<tr>
<td>6</td>
<td>Numeric and geometric patterns</td>
<td>88</td>
</tr>
<tr>
<td>7</td>
<td>Functions and relationships</td>
<td>105</td>
</tr>
<tr>
<td>8</td>
<td>Algebraic expressions</td>
<td>115</td>
</tr>
<tr>
<td>9</td>
<td>Equations</td>
<td>133</td>
</tr>
<tr>
<td>10</td>
<td>Revision</td>
<td>146</td>
</tr>
</tbody>
</table>

### Term 2

<table>
<thead>
<tr>
<th>Unit</th>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>Constructions</td>
<td>156</td>
</tr>
<tr>
<td>12</td>
<td>Investigating properties of geometric figures by construction</td>
<td>166</td>
</tr>
<tr>
<td>13</td>
<td>Classifying 2-D shapes</td>
<td>180</td>
</tr>
<tr>
<td>14</td>
<td>Similar and congruent triangles</td>
<td>192</td>
</tr>
<tr>
<td>15</td>
<td>Geometry of straight lines</td>
<td>207</td>
</tr>
<tr>
<td>16</td>
<td>The Theorem of Pythagoras</td>
<td>223</td>
</tr>
<tr>
<td>17</td>
<td>Area and perimeter</td>
<td>237</td>
</tr>
<tr>
<td>18</td>
<td>Revision</td>
<td>259</td>
</tr>
</tbody>
</table>

### Term 3

<table>
<thead>
<tr>
<th>Unit</th>
<th>Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>Functions and relationships</td>
<td>267</td>
</tr>
<tr>
<td>20</td>
<td>Algebraic expressions</td>
<td>277</td>
</tr>
<tr>
<td>21</td>
<td>Equations</td>
<td>293</td>
</tr>
<tr>
<td>22</td>
<td>Graphs – Part 1</td>
<td>305</td>
</tr>
<tr>
<td>23</td>
<td>Graphs – Part 2</td>
<td>319</td>
</tr>
<tr>
<td>24</td>
<td>Surface area and volume</td>
<td>331</td>
</tr>
<tr>
<td>25</td>
<td>Revision</td>
<td>359</td>
</tr>
</tbody>
</table>
Term 4

Unit 26  Transformation geometry ................................................................. 366
Unit 27  Geometry of 3-D objects ................................................................. 391
Unit 28  Collect, organise and summarise data ........................................... 409
Unit 29  Representing data ........................................................................... 425
Unit 30  Analyse, interpret and report data .................................................. 439
Unit 31  Probability ....................................................................................... 457
Unit 32  Revision ......................................................................................... 471

Glossary ........................................................................................................... 482

Questions and activities purposed for enrichment are indicated using this icon. Use these questions and activities to challenge learners’ thinking about a concept being learnt.
In this unit you will:

- revise Grade 8 operations on whole numbers
- describe the **real number system** as consisting of various types of numbers
- recognise the following types of numbers in the real number system:
  - natural numbers (excludes zero)
  - whole numbers (includes zero)
  - integers
  - rational numbers
  - irrational numbers
- perform basic operations on numbers from the real number system in order to:
  - calculate exact answers
  - estimate answers by rounding off
  - practise various calculation techniques
- factorise natural numbers by using prime factorisation in order to find the LCM and HCF
- apply your knowledge of the real number system and the basic operations to solve problems involving:
  - ratio and rate
  - direct and inverse proportion
- explore financial contexts.

Getting started

1. Mental calculations: calculate the following ‘in your head’ only:
   - a) $8 \times 8$
   - b) $11 \times 7$
   - c) $3 \times 9$
   - d) $2\,115 \times 2$
   - e) $5 \times 7$
   - f) $7 \times 4$
   - g) $77 \times 2$
   - h) $12 \times 8$

2. Calculations with whole numbers: use the column method to multiply, add and subtract:
   - a) $654 \times 23$
   - b) $1\,342 \times 47$
   - c) $9\,351 \times 0$
   - d) $102\,987 + 534\,675$
   - e) $898\,564 + 312\,342$
   - f) $11\,981 + 0$
   - g) $7\,586 - 354$

3. Use long division to calculate:
   - a) $5\,210 \div 13$
   - b) $15\,428 \div 532$
   - c) $0 \div 45\,439$
   - d) $3\,471 \div 0$
4. Estimating: round off the following numbers to the nearest 10 and then estimate (without using a calculator) the result of the following operations. State whether it is an underestimate or overestimate, and then check the actual answer by using a calculator.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Estimate</th>
<th>Underestimate/overestimate?</th>
<th>Answer on calculator (two decimal places):</th>
</tr>
</thead>
<tbody>
<tr>
<td>$719 \div 11$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$103 - 24$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$77 \div 37$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$123 \times 19$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$348 + 33$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Round up or down to an appropriate value in order to compensate for rounding in the following mental calculations. You may then use a calculator to check how close your estimate is to the actual answer. The first one has been done for you.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Estimate</th>
<th>Checked against answer on calculator (rounded to two decimal places)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$36 \div 17$</td>
<td>40 \div 20 = 2</td>
<td>2.12</td>
</tr>
<tr>
<td>$87 \times 9$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$76 \div 37$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$124 \times 17$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$458 + 93$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$1015 - 717$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key ideas

- It is useful to round off in a way that reduces the magnitude of overestimation or underestimation. This is called compensating for the rounding error.
- In general, remember the following ideas for compensation. They are related to the operation being used:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Remember ...</th>
</tr>
</thead>
</table>
| $\div$    | - If the dividend is rounded up, then round up the divisor by an appropriate amount e.g. $36 \div 17$, then 36 rounded up to 40 and 17 rounded up to 20; hence we have $40 \div 20$.
- If the dividend is rounded down, then round down the divisor by an appropriate amount.
Activity 1.1  Column addition and multiplication

1. Add the two numbers 15 455 and 9 600 and then check your answer against the one given:

```
  1 5 4 5 5
+ 9 6 0 0
```

```
  2 5 0 5 5
```

2. In the hundreds column, we had 4 + 6, yet we wrote 0 as its answer. Explain why we didn’t write 10.

3. Here are two numbers for you to add using the column addition method:

```
2 7 8 6 9
+ 5 6 1 3
```

```
1 2 4 3 8
```

```
2 7 8 6 9
+ 5 6 1 3
```

```
1 2 4 3 8
```

4. Mr Matabane gives his Grade 9 class the multiplication problem 159 × 27.

**Mthunzi writes:**

\[
27 \times 159 = (20 \times 159) + (7 \times 159) \\
= 20 \times (100 + 50 + 9) + 7 \times (100 + 50 + 9) \\
= (20 \times 100) + (20 \times 50) + (20 \times 9) + (7 \times 100) + (7 \times 50) + (7 \times 9) \\
= 2000 + 1000 + 180 + 700 + 350 + 63 \\
= 3180 + 1113 \\
= 4293
\]

**Katie writes:**

\[
\begin{array}{c}
1 5 9 \\
\times 2 7 \\
\hline \\
1 1 1 3 \\
+ 3 1 8 0 \\
\hline \\
4 2 9 3 \\
\end{array}
\]

Explain Mthunzi’s and Katie’s methods.
5. Explain the long division method used to divide 25 891 by 17, as shown below:

![Long Division Example]

Key ideas

Here we revise methods that are familiar to you:

- The column addition method arranges the numbers vertically one below the other so that all the units are in the same column, all the tens in the next, all the hundreds in the 3rd, etc. This simplifies the addition of numbers that have 3 or more digits (if the calculation is done without a calculator).
- The column multiplication and long division has the same purpose – to make pencil-and-paper calculations easier. With a calculator, these methods are tiresome. It may also be that you could do these mentally and not need the methods revised.

Exercise 1.1

1. Add 11 384 + 8 956, using column addition.
2. Multiply 586 by 36, using the column multiplication method.
3. Divide 8 448 by 16, using long division.
4. Multiply 786 by 19 using column multiplication.
5. a) Divide 2 013 by 13 and give the remainder as well.
   b) Write your answer as a composite number – whole number and fraction.
   c) Write your answer in decimal form (up to 2 places after the comma) – that is, write the fraction part as a decimal after the comma.
Activity 1.2  Natural numbers

Read the following and answer the questions that follow:

• The most simple counting method is that of using our fingers.

• As a result, most societies have the number ‘10’ represented in their numeral systems.

• The base-10 numeral system is the most commonly used way to represent numbers.

• Using base-10 and counting our fingers, we get the first subset of the natural numbers, which we may represent in the following way:
  First ten natural numbers = {1; 2; 3; 4; 5; 6; 7; 8; 9; 10}

• We can count beyond ten and never reach a ‘last number’ where our counting comes to an end. For this reason we represent the natural numbers as:
  \( \mathbb{N} = \{1; 2; 3; \ldots\} \), or simply just as \( \mathbb{N} \)

Note the three dots, or ellipsis, which indicate that the numbers continue forever.

• Sometimes we include 0 in the set of natural numbers and write \( \mathbb{N} = \{0; 1; 2; 3; \ldots\} \)

• We can perform the basic operations (+, −, ×, ÷) on the natural numbers, but the answers may or may not be natural numbers.

Answer these questions by choosing the correct answer:

1. Which of the following numbers is a natural number?
   A. 3,06  B. 5  C. \( 7\frac{1}{2} \)  D. 0

2. The number −3 is a:
   A. natural  B. whole  C. integer  D. irrational

3. Which of the following numbers is a natural number?
   A. \( \sqrt{4} \)  B. \( \sqrt{5} \)  C. \( \frac{3}{4} \)  D. \( \frac{3}{9} \)

4. Which of the following answers are natural numbers?
   A. \( 59 \div 7 = \square \)  B. \( 58 \div 7 = \square \)  C. \( 57 \div 7 = \square \)  D. \( 56 \div 7 = \square \)

5. Which of the following answers are natural numbers?
   A. \( 7 897 \div 7 = \square \)  B. \( 7 788 - 7 988 = \square \)
   C. \( 957,3 \times 7 = \square \)  D. \( -64 + 72 = \square \)
Activity 1.3  Identifying natural numbers

Complete the table by doing the operations indicated. In the last column, state whether or not the answer to your calculations may be found in the set of natural numbers. The first two have been completed for you.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Natural numbers</th>
<th>Answer</th>
<th>Is it in the set ( \mathbb{N} )?</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ (8; 5)</td>
<td>8 + 5 = 13</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>– (4; 9)</td>
<td>4 – 9 = –5</td>
<td>No</td>
<td></td>
</tr>
<tr>
<td>+ (13; 7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– (14; 9)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>× (5; 8)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>÷ (4; 2)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>– (7; 7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>÷ (3; 6)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key ideas

- Some of your answers in Activity 1.3 do not form part of the set of natural numbers, (i.e. negatives and fractions).
- We must, therefore, extend the set of natural numbers to a bigger set that includes negatives and fractions.
- Extensions of the set of natural numbers \( (\mathbb{N}) \) may be summarised as follows:

<table>
<thead>
<tr>
<th>Extension to include …</th>
<th>Name of set formed</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negatives</td>
<td>Integers</td>
<td>( \mathbb{Z} )</td>
</tr>
<tr>
<td>Fractions</td>
<td>Rational numbers</td>
<td>( \mathbb{Q} )</td>
</tr>
</tbody>
</table>

Integers extend the whole number and natural number systems so that a bigger number can be subtracted from a smaller number, e.g. \( 3 – 5 = –2 \). In algebraic terms, integers include the operation \( a – b \), where \( a < b \).

- Rational numbers are defined as numbers that can be written in the form \( \frac{a}{b} \), where \( a \) and \( b \) are integers, and \( b \neq 0 \).
- When extracting roots, such as square roots \( \sqrt{\_} \) of non-perfect squares and cube roots \( \sqrt[3]{\_} \) of non-perfect cubes, we get answers which are irrational \( (\mathbb{Q}') \).
- The number \( \pi \) is another example of an irrational number.
Activity 1.4  Natural numbers

\[ \mathbb{N} = \{1; 2; 3; \ldots \} \] where \( \mathbb{N} \) stands for the set or collection of all natural numbers, of which some examples are \( \{1; 2; 3; \ldots \} \). The three dots ‘...’ mean that the numbers continue forever (until infinity) by adding one each time.

Sometimes the natural numbers are represented on a number line, as shown below.

Since you have seen and worked with these numbers often, see if you can answer the questions that follow.

Discuss whether the following statements are true or false.

Statement 1: When you add any two natural numbers, your answer is always another natural number.

Statement 2: When you multiply any two natural numbers, your answer is always another natural number.

Statement 3: Whenever you subtract two natural numbers your answer is always another natural number.

Statement 4: Whenever you divide two natural numbers, your answer is always a natural number.

Key ideas

- For any given natural number, the next one is obtained by the addition of 1. For example, the number that follows 23 is \( 23 + 1 = 24 \). This also means that the natural numbers go on forever because you can always add 1 to continue.
- *Consecutive* natural numbers lie next to each other; 3, 4, 5 or 26, 27, 28 are consecutive numbers.
- By looking at some examples we see that the sum or product of any two natural numbers produces another natural number, for example, \( 2 + 5 = 7 \) and \( 2 \times 5 = 10 \) (2 and 5 are natural numbers, and so are 7 and 10).
- However, for subtraction, this is not always the case. For example, \( 5 - 2 = 3 \), a natural number, but \( 2 - 5 = -3 \), which is NOT a natural number, because natural numbers are only positive whole numbers.
- Similarly, when we divide two natural numbers, we do not always get an answer from the natural number set, e.g. \( 3 \div 2 \) does not give us a positive whole number, so the answer is not a natural number.
Exercise 1.2

1. Say whether the following lists of numbers are consecutive natural numbers or not. Explain your answers.
   a) 112, 113, 114
   b) 2, 4, 6
   c) \(n - 1, n, n + 1\) where \(n\) is any natural number bigger than 2.
   d) 10, 20, 30
   e) 81, 80, 79

2. You are given the natural numbers 5 and 15.
   a) Is the sum of the two numbers a natural number? If so, what is it?
   b) Subtract the two numbers so that the answer will be:
      i) A natural number.
      ii) Not a natural number.
   c) When you multiply the two numbers 5 and 15, will the product be a natural number?
   d) When you divide the two numbers, will the answer always be a natural number?

Activity 1.5  Natural numbers to integers

We can use the set of natural numbers to count with, and to perform any addition or multiplication calculation. The answers to these calculations will always still be natural numbers. As for subtraction, as long as we make sure that we always subtract the smaller from the bigger number, we will also have natural numbers as answers; for example, the answer to \(5 - 2\) can be represented by the natural number 3. However, we cannot represent the answer to \(2 - 5\) with a natural number.

One way of solving this problem is by extending the set of natural numbers to include numbers which would represent ‘answers’ to expressions like \(2 - 5\). These numbers are a little different from the way we think of the numbers that we count with. These numbers are zero and numbers ‘less than 0’.

1. If we now extend the natural numbers to include zero and numbers ‘less than zero’, we have a new set of numbers that includes all the negative whole numbers, as well as the positive whole numbers, along with 0 itself. This set is called the integers.

Discuss and compare your answers to the following questions in class.
a) With the natural numbers you saw that when you subtract any two natural numbers, the answer is NOT always a natural number. Does introducing the set of integers solve this problem? If you subtract any two integers, will the answer ALWAYS be another integer? Use examples to explain your answer.

b) When you multiply integers, the answer is always another integer. Check this with some examples.

2. Do the integers have any beginning or any end? Are there any biggest or smallest integers? Explain your answer.

3. With the addition, subtraction and multiplication of integers, the answers are still integers, but what happens when you divide integers? When you divide any two non-zero integers, is the answer ALWAYS another integer? Use examples to explain your answer.

• The set of integers includes all the positive and negative whole numbers and 0.

![Integer Number Line]

The integers continue in both directions forever.

• When we add, subtract or multiply integers, the answers are always integers. This is important because it means that we can perform any calculation involving addition, subtraction or multiplication and ALWAYS be able to represent the answer as an integer.

• Because the integers include negative and positive numbers, we also have to look again at what the addition and multiplication of integers mean. The ‘rules’ for multiplying integers are given in the table:

<table>
<thead>
<tr>
<th></th>
<th>+</th>
<th>−</th>
<th>+</th>
<th>−</th>
</tr>
</thead>
<tbody>
<tr>
<td>+m</td>
<td>×</td>
<td>+n</td>
<td>=</td>
<td>+mn</td>
</tr>
<tr>
<td>−m</td>
<td>×</td>
<td>−n</td>
<td>=</td>
<td>+mn</td>
</tr>
<tr>
<td>+m</td>
<td>×</td>
<td>−n</td>
<td>=</td>
<td>−mn</td>
</tr>
<tr>
<td>−m</td>
<td>×</td>
<td>+n</td>
<td>=</td>
<td>−mn</td>
</tr>
</tbody>
</table>
Activity 1.6  Rational numbers from integers

Because fractions arise from the division of two integers, we can define fractions as numbers that can be written as a division or ratio of two integers, e.g. $3 \div 4$ and $3 : 4$ can be represented as $\frac{3}{4}$. Even whole numbers can be written as fractions. For example, we can write the whole number 2 as a division of two integers, $2 \div 1$, and in fraction notation as $\frac{2}{1}$.

Discuss the following questions in class.

1. Is there always a fraction between any two fractions? Give reasons for your answer.
2. How many fractions are there between 0 and 1?
3. What fraction lies just before 1? Explain your answer or show what you mean on a number line.

Key ideas

- The number set that includes all the integers and all the fractions between the integers, is called the set of rational numbers.

**Definition 1.1.** Rational numbers can be written in the form $\frac{a}{b}$, where $a$ and $b$ are both integers and $b \neq 0$.

- The form $\frac{a}{b}$ represents the division of two integers or the ratio of two integers. Thus rational numbers can also be written as $a : b$, where $a$ and $b$ are integers and $b \neq 0$.

- If the integers have no beginning or end, the same should be true for the rational numbers because the set of rational numbers includes the integers.

- You can always find fractions between any two fractions; for example, between the fractions $\frac{1}{2}$ and $\frac{3}{4}$ are $\frac{5}{8}$, $\frac{9}{16}$, $\frac{17}{32}$, etc. and many, many more. For this reason there is no rational number ‘right next to’ another rational number.

- Rational numbers can be made as small as you like simply by increasing their denominators – for example, the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc. decrease in size as the denominators increase in size.

**Definition 1.2.** Numbers (like $\sqrt{2}$, $\sqrt{3}$ and $\pi$), which cannot be written as a ratio of integers and which have a non-repeating infinite sequence of digits when written out in decimal notation, are called irrational numbers.
• If you attempt to calculate \( \sqrt{2} \) by using an electronic calculator you get an answer showing a finite number of decimal places, giving you a *rational* number approximation to \( \sqrt{2} \).

![Diagram showing \( \sqrt{2} \) units and 1 unit]

**Definition 1.3.** The set of numbers consisting of the rational numbers (which includes the integers) and the irrational numbers is called the *real numbers*.

### Activity 1.7  Properties of numbers

The real numbers \( \mathbb{R} \) (which includes the rational and irrational numbers) with the operations, addition (+) and multiplication have the following properties:

**Property 1:** For addition and multiplication: If \( a \) and \( b \) are real numbers, then

- \( a + b \) is a real number
- \( a \times b \) is a real number

**Property 2:** For addition the real numbers has *identity* element, 0 and for multiplication the *identity* is 1. This means that if \( a \) is any real number,

\[
\begin{align*}
    a + 0 &= 0 + a = a, \\
    a \times 1 &= 1 \times a = a.
\end{align*}
\]

**Property 3:** The operations addition and multiplication (of real numbers) are *associative*; this means that for any real numbers \( a, b \) and \( c \),

\[
\begin{align*}
    (a + b) + c &= a + (b + c) \\
    (a \times b) \times c &= a \times (b \times c)
\end{align*}
\]

**Property 4:** Addition and multiplication of real numbers are *commutative*; that is, for any two real numbers \( a \) and \( b \),

\[
\begin{align*}
    a + b &= b + a, \\
    a \times b &= b \times a.
\end{align*}
\]

**Property 5:** Multiplication is *distributive* over addition (or subtraction); this means that for natural numbers \( a, b \) and \( c \),

\[
\begin{align*}
    a \times (b + c) &= (a \times b) + (a \times c) \\
    a \times (b - c) &= (a \times b) - (a \times c)
\end{align*}
\]
Property 6: Every real number has an additive inverse:
If \( a \) is a real number, then the additive inverse of \( a \) is \(-a\); notice \( a + (-a) = 0 \) (identity for addition)

The multiplicative inverse of \( a \) is \( \frac{1}{a} \), for \( a \neq 0 \); notice \( a \times \frac{1}{a} = 1 \) (identity for multiplication)

1. Some of the properties like Property 1 work for subtraction and division as well. We do have to take care with operations involving 0. What difficulty does division by zero produce?

2. Check if Property 2 works for subtraction and division – if not, give an example to show what you mean.

3. What is the additive inverse of (a) 5 and (b) \(-9\)?

4. What is the multiplicative inverse of (a) 5 and (b) \(-\frac{3}{4}\)?

5. Why does 0 not have a multiplicative inverse?

Key ideas

- The multiplicative inverse of 0 should be a real number that must produce 1 (the identity) when multiplied by 0. There is no such number.
- These basic properties of real numbers allow us to do almost all of the calculations and algebra in high school mathematics.

Exercise 1.3

1. The next set of questions explore some of the properties of numbers we have investigated.
   a) For rational numbers, it seems that whether you add, multiply, subtract or divide any two rational numbers, the answer is always another rational number. Do you agree? If you don’t, then try to find two rational numbers where this is not the case. If you do agree with the statement, then write out some examples involving addition, multiplication, subtraction and division of rational numbers to show that the answers are always rational too.
   b) Explain why there is a need to include irrational numbers in our number sets.
2. The ratio between the circumference \( C \) of a circle and its diameter \( D \), is \( \pi \). JH Lambert (1728–1777) proved that the number \( \pi \) is irrational. This means that no matter how many millions of decimal digits we use, \( \pi \) can never have an exact decimal fraction representation.

Through the ages various approximations have been used for \( \pi \). From the table below you can also see some historical development in rational approximations for \( \pi \).

Note that even the last entry (the 21 digit decimal fraction) is not the precise value of \( \pi \).

<table>
<thead>
<tr>
<th>Period</th>
<th>People/Source</th>
<th>Value for ( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2000 BC</td>
<td>Egyptians</td>
<td>3,16</td>
</tr>
<tr>
<td>1100 BC</td>
<td>Chinese</td>
<td>3</td>
</tr>
<tr>
<td>300 BC</td>
<td>Archimedes (Greece)</td>
<td>Between ( \frac{210}{71} ) and ( \frac{7}{3} )</td>
</tr>
<tr>
<td>530 AD</td>
<td>India</td>
<td>3,1416; sometimes ( \sqrt{10} )</td>
</tr>
<tr>
<td></td>
<td>Calculator to 10 digits</td>
<td>3,141592654</td>
</tr>
<tr>
<td></td>
<td>Up to 21 digits</td>
<td>3,14159265358979323846</td>
</tr>
</tbody>
</table>

If someone says \( \pi \) is \( \frac{22}{7} \), why is this incorrect?

3. Between which of the two integers does \( \sqrt{11} \) lie such that there will be no other integers in-between your chosen ones?

4. Explain why \( \sqrt{99} \) lies between 9 and 10.

5. Which is a better approximation of \( \sqrt{8} \)
   a) 2 or 3?
   b) 2,8 or 2,9?
Exercise 1.4  Identifying numbers from different number systems

- Copy the following table into your notebook.
- For each value in the first column, indicate with a tick (✓) to which set it belongs. Use a cross (✗) to indicate that it does not belong to a set. Round your answers to three decimal places where appropriate.
- Recall the order of operations: BODMAS.

<table>
<thead>
<tr>
<th>Value</th>
<th>ℤ</th>
<th>ℚ</th>
<th>ℚ'</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 − (11 + 4) =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 + 15 =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21 ÷ (5 − 2) =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 × (55 ÷ 5) =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>√5 =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(11 + 8) − (34 − 15) =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(11 − 4) / (7,3) =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>π =</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22 / 7 =</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Key ideas

- All the number systems mentioned collectively form the real number system (denoted by the capital letter \( \mathbb{R} \)).
- The real number system may therefore be represented as:

REAL \( \mathbb{R} \)  
Rational \( \mathbb{Q} \)  
Integers \( \mathbb{Z} \)  
Natural \( \mathbb{N} \)  
Irrational \( \mathbb{Q}' \)
Activity 1.8  Prime decomposition

1. Complete the following table in order to factorise the composite numbers into prime factors:

<table>
<thead>
<tr>
<th>Composite number</th>
<th>Prime factors</th>
<th>Rewrite in factor form</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>2; 2; 3; 5</td>
<td>60 = 2^2.3.5</td>
</tr>
<tr>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>539</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 520</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Find all the prime factors of the following numbers:
   a) 36
   b) 21
   c) 7
   d) 20
   e) 11
   f) 921
   g) 30

3. For each of the following:
   i) fill in the missing factors
   ii) circle the prime factors.
   a) The factors of 24 are 1; 2; □; 4; □; 8; □; 24.
   b) The factors of 36 are 1; 2; □; □; □; 9; □; □; 36.
   c) The factors of 144 are □; □; □; □; □; 12; 16; 18; 24; 36; 48; 72; 144.

4. Copy the following grid into your notebook. Shade the prime numbers to see if you can find a path from the top to the bottom of the grid.

<table>
<thead>
<tr>
<th>46</th>
<th>40</th>
<th>51</th>
<th>83</th>
<th>87</th>
<th>135</th>
<th>6</th>
<th>44</th>
</tr>
</thead>
<tbody>
<tr>
<td>36</td>
<td>13</td>
<td>125</td>
<td>126</td>
<td>1</td>
<td>61</td>
<td>72</td>
<td>127</td>
</tr>
<tr>
<td>136</td>
<td>91</td>
<td>83</td>
<td>122</td>
<td>120</td>
<td>87</td>
<td>107</td>
<td>51</td>
</tr>
<tr>
<td>74</td>
<td>94</td>
<td>119</td>
<td>149</td>
<td>71</td>
<td>117</td>
<td>109</td>
<td>29</td>
</tr>
<tr>
<td>148</td>
<td>59</td>
<td>93</td>
<td>96</td>
<td>139</td>
<td>49</td>
<td>87</td>
<td>7</td>
</tr>
<tr>
<td>39</td>
<td>29</td>
<td>43</td>
<td>115</td>
<td>129</td>
<td>64</td>
<td>9</td>
<td>46</td>
</tr>
<tr>
<td>98</td>
<td>65</td>
<td>15</td>
<td>144</td>
<td>66</td>
<td>127</td>
<td>139</td>
<td>137</td>
</tr>
<tr>
<td>130</td>
<td>60</td>
<td>53</td>
<td>110</td>
<td>60</td>
<td>37</td>
<td>120</td>
<td>19</td>
</tr>
</tbody>
</table>
Key ideas

- Every positive integer, apart from 1, has a unique set of prime number factors: this is the Fundamental Theorem of Arithmetic, aka the Unique Factorisation Theorem.
- Prime numbers are positive integers (excluding 1) which have only two factors: themselves and 1. The first five primes are 2; 3; 5; 7; 11.
- Numbers which are not prime numbers are called composite numbers: they have more than two factors. The first five composite numbers are 4; 6; 8; 9; 10.
- It is fairly easy to find prime factors for even numbers since we know we can always start with the prime number 2 as a factor (e.g. 60; 42; 2 520).
- Similarly, if the composite number ends with a 5 or a zero, then the prime number 5 must be a factor (e.g. 60; 945; 2 520).
- The composite number 539 is neither a multiple of 2 nor a multiple of 5. Here it is useful to remember the divisibility rules for the primes:

<table>
<thead>
<tr>
<th>Divisible by ...</th>
<th>If ...</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>The number is even</td>
<td>2; 4; 6; 8; ...; 60; ...; 378; ...</td>
</tr>
</tbody>
</table>
| 3                | The sum of the digits is a multiple of three | 60: 6 + 0 = 6  
57: 5 + 7 = 12  
144: 1 + 4 + 4 = 9 |
| 5                | The last digit is zero or 5 | 10; 35; 105; 970; 3 795 |

- Divisibility rules for prime factors 7 and 11 are more complex. In Grade 9 it is sufficient to use trial-and-improvement to find the factors.
- Two methods are useful for finding prime factors: the factor tree and the ladder method.

Worked example

Prime factorise the composite number 1 485.

Factor tree

We note that the last digit is a 5, and therefore we may start with 5 as a prime factor:

```
1 485
 5 297
  3 99
   3 33
    3 11
```

So 1 485 = 5 × 3 × 3 × 3 × 11
Ladder method

\[
\begin{array}{c|c}
5 & 1485 \\
3 & 297 \\
3 & 99 \\
3 & 33 \\
11 & 11 \\
1 & \\
\end{array}
\]

(divide 5 into the number)

(sum of digits is multiple of 3)

Note: You can use any method to find prime factors. The methods given are merely suggestions.

Exercise 1.5 Prime factorisation

Find the prime factors of the following composite numbers. Use a method of your choice.

1. 50
2. 294
3. 539
4. 5544
5. 7800
6. 1729

Activity 1.9 HCF and LCM

Copy and complete the following:

1. HCF stands for H____ C____ Factor. The HCF is the biggest ____ that will divide without a ____ into two or more numbers.
   a) ____ is the HCF of 12 and 18.
   b) ____ is the highest common factor of 52 and 13.

2. LCM stands for the L____ Common M____. The LCM is the ____ number that is a ____ of two or more numbers.
   a) ____ is the LCM of 6 and 8.
   b) ____ is the lowest common multiple of 7 and 14.

Worked example

Find the HCF and LCM of the following composite numbers:

a) 12 and 8
b) 168 and 252

Hint:
• A useful technique for large numbers is to use prime factorisation.
• From the prime factors of each composite number, we then extract the HCF and LCM.
a) The factors of 12 are: \( F_{12} = \{1; 2; 3; 4; 6; 12\} \)
The factors of 8 are: \( F_8 = \{1; 2; 4; 8\} \)
By comparing the sets, we see that the HCF = 4.
The LCM is a multiple of the HCF.

b) Listing the factors for large numbers is not practical.
We will use the following alternative:

\[
\begin{array}{c|ccc}
2 & 168 & 252 \\
2 & 84 & 126 \\
3 & 42 & 63 \\
7 & 14 & 21 \\
2 & 3 & \end{array}
\]
The HCF is then \(2 \times 2 \times 3 \times 7 = 84\).

**Exercise 1.6**  
**HCF and LCM**

Find the HCF and LCM of the following groups of numbers:

1. 18; 48 and 54
2. 385 and 770
3. 648 and 864
4. 1 029 and 2 058
5. 1 080 and 1 215

**Activity 1.10**  
**Ratio and rate**

1. Simplify the given ratios:
   a) 18 : 27  
   b) 12 : 18 : 24
2. Tessa and Tsengiwe have 32 sweets that they wish to share between them in the ratio 3 : 5. Calculate how many sweets each person gets.
3. A music store has a sale in which the CDs are reduced in the ratio 3 : 2. Calculate the cost of a CD which originally cost:
   a) R99  
   b) 199
4. The scale distance on a map of South Africa is 1 : 50 000. What is the actual distance between two towns if the distance on the map is 5 cm?
5. Bananas are sold at R7,99/kilogram. Calculate the mass of bananas that you could buy for R20.
6. If a car travels 15 kilometres in 15 minutes, calculate how far the car would travel if it travelled at the same average speed for 1 h 10 min.
Key ideas

- A ratio is a comparison of two or more quantities of the same kind, such as the volumes of two liquids that may be mixed together.
- The HCF may be used to simplify a ratio.
- A rate is a comparison of two different quantities.
- Speed is an example of a rate, which compares the distance travelled to the time taken to travel the distance. The equation relating these quantities is:
  \[
  \text{Speed} = \frac{\text{distance}}{\text{time}} \quad \text{Time} = \frac{\text{distance}}{\text{speed}} \quad \text{Distance} = \text{speed} \times \text{time}
  \]
- The formula for speed calculates the average speed.

Exercise 1.7  Ratio and rate

1. Rewrite the following ratios in their simplest form:
   a) 4 : 24  b) 33 : 77  c) 32 : 64 : 96  d) 63 : 84 : 105

2. John and Bianca are challenging each other at computer games and decide to share a huge block of chocolate as the prize. They share the chocolate in the ratio of how many games each person has won. Calculate how much of the chocolate each one gets if it is made of 60 smaller blocks and if John wins seven games and Bianca wins five games.

3. The scale on a plan for a house is 1 : 30. Calculate the actual dimensions of a room which measures 17,25 cm by 15,79 cm by 10 cm on the plan. Round off your answers to the nearest centimetre.

4. Use the formulae provided to calculate the following. Use appropriate units for your answers and round off to one decimal place.
   a) The average speed of a train that travels in \(4 \frac{1}{2}\) minutes between two stations which are 5,2 km from each other.
   b) The speed of light in a vacuum is about 300 000 km per second. Calculate in minutes how long it takes for sunlight to reach:
      i) Earth, which is about 150 000 000 km from the sun
      ii) Mars, which is about 230 000 000 km from the sun.
   c) A javelin thrower at the Olympic Games launches a javelin at a speed of 83 km/h. How far did the Olympian throw the javelin if it remained above ground for four seconds?
5. A jet plane descends from a height of 10 000 metres to land within 45 minutes.
   a) Calculate the rate of descent in km/h (to the nearest whole number rate).
   b) How long would it take to descend at the same rate from 12 000 metres? Give your answer to the nearest minute.

**Activity 1.11  Proportion**

Check to see whether the following tables represent values which are in direct proportion or inverse proportion or neither. Give a short reason for your answer.

1. \[
\begin{array}{c|c|c|c|c}
\text{Number of litres of gas (t)} & 3 & 4 & 5 & 6 \\
\text{Empty space in the tank (s)} & 4 & 3 & 2.4 & 2 \\
\end{array}
\]

2. \[
\begin{array}{c|c|c|c|c}
\text{Distance travelled (kilometres)} & 20 & 60 & 100 & 120 \\
\text{Time taken (minutes)} & 15 & 45 & 75 & 90 \\
\end{array}
\]

3. \[
\begin{array}{c|c|c|c|c}
\text{Time spent on Mxit (minutes)} & 30 & 60 & 75 & 120 \\
\text{Number of new ‘friends’} & 2 & 4 & 5 & 6 \\
\end{array}
\]

**Key ideas**

- Two quantities are said to be in **direct proportion** if the ratio between corresponding values is constant and as one value increases, the other increases at a fixed rate; for example, the cost of filling a car with petrol increases at a fixed rate. The constant ratio between the cost and number of litres is the unit price per litre of petrol.
  - The equation which represents direct proportion is \( \frac{y}{x} = k \), where \( k \) is a constant.

- Two quantities are in **inverse proportion** if the product between corresponding values is constant and as one value increases, the other decreases; for example, on a hot day evaporation causes the water level to drop in a water tank. As time passes during the day, the water level gets lower in the tank.
  - The equation which represents inverse proportion is \( x \cdot y = k \), where \( k \) is a constant.
  - **Note:** Inverse proportion is sometimes referred to as **indirect proportion**.
## Exercise 1.8 Proportion

### 1. Check whether the following tables contain values in direct or inverse proportion or neither. Fill in the missing values for all the tables.

#### a)

<table>
<thead>
<tr>
<th>Distance (km)</th>
<th>70</th>
<th>140</th>
<th>210</th>
<th>280</th>
<th>770</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

#### b)

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Breadth (m)</td>
<td>18</td>
<td>12</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>

#### c)

<table>
<thead>
<tr>
<th>Speed (km/h)</th>
<th>120</th>
<th>80</th>
<th>40</th>
<th>30</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (h)</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

#### d)

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed (m/s)</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
</tr>
</tbody>
</table>

#### e)

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>21</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>56</td>
</tr>
</tbody>
</table>

### 2. A photograph is 35 mm long by 28 mm wide. The width of the enlargement is 14 cm. What is its length?

### 3. It takes 60 workers six days to complete a certain length of road. If only 30 workers do the job, it takes 12 days to complete the same length of road.

#### a)

<table>
<thead>
<tr>
<th>Workforce</th>
<th>60</th>
<th>40</th>
<th>30</th>
<th>20</th>
<th>15</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Job time</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

#### b)

Describe the relation between the time taken and the number of workers required to complete the job.

#### c)

Complete the following equation: Job time = \[
\frac{\text{workforce}}{\text{workforce}}
\]

#### d)

Use your equation to calculate how many days it would take 35 workers to do the job. **Note:** Each fraction of a day is taken as a whole day.
Activity 1.12  Simple interest and compound interest

An investor wants to invest R8 000 in an interest-bearing account for five years. There is a choice between two accounts: Account A offers 13% simple interest per annum, whereas Account B offers compound interest at 13% per annum. Which account offers the best return over
1. one year  2. three years  3. five years?

Note: We can answer this question by doing repeated calculations, one for each of the years of the investment.

Copy the table below and complete the calculations in order to answer the question.

<table>
<thead>
<tr>
<th>Simple interest at 13% p.a.</th>
<th>Compound interest at 13% p.a.</th>
</tr>
</thead>
<tbody>
<tr>
<td>End of year 1</td>
<td>End of year 1</td>
</tr>
<tr>
<td>End of year 2</td>
<td>End of year 2</td>
</tr>
<tr>
<td>End of year 3</td>
<td>End of year 3</td>
</tr>
<tr>
<td>End of year 4</td>
<td>End of year 4</td>
</tr>
<tr>
<td>End of year 5</td>
<td>End of year 5</td>
</tr>
</tbody>
</table>

Worked example
1. Calculate the value of an investment of R2 000 after four years at 15% p.a., simple interest. Use repeated calculations.

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount in investment (R)</th>
<th>+ Interest earned</th>
<th>= Total at end of year (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 000</td>
<td>+ 2 000 \times \frac{15}{100}</td>
<td>= 2 300,00</td>
</tr>
<tr>
<td>2</td>
<td>2 300</td>
<td>+ 2 000 \times \frac{15}{100}</td>
<td>= 2 600,00</td>
</tr>
<tr>
<td>3</td>
<td>2 600</td>
<td>+ 2 000 \times \frac{15}{100}</td>
<td>= 2 900,00</td>
</tr>
<tr>
<td>4</td>
<td>2 900</td>
<td>+ 2 000 \times \frac{15}{100}</td>
<td>= 3 200,00</td>
</tr>
</tbody>
</table>

2. Calculate the value of an investment of R2 000 after four years at 15% p.a., compound interest.

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount in investment (R)</th>
<th>+ Interest earned</th>
<th>= Total at end of year (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 000,00</td>
<td>+ 2 000 \times \frac{15}{100}</td>
<td>= 2 300,00</td>
</tr>
</tbody>
</table>
### Yearly Amounts of Investment

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount in investment (R)</th>
<th>+</th>
<th>Interest earned</th>
<th>=</th>
<th>Total at end of year (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2 300,00</td>
<td>+</td>
<td>2 300 × \frac{15}{100}</td>
<td>=</td>
<td>2 645,00</td>
</tr>
<tr>
<td>3</td>
<td>2 645,00</td>
<td>+</td>
<td>2 645 × \frac{15}{100}</td>
<td>=</td>
<td>3 041,75</td>
</tr>
<tr>
<td>4</td>
<td>3 041,75</td>
<td>+</td>
<td>3 041,75 × \frac{15}{100}</td>
<td>=</td>
<td>3 498,0125</td>
</tr>
</tbody>
</table>

**Note:** Rounding off is applied only to the final answer – R3 498.01.

### Key ideas

- **When doing calculations with simple interest,** we add the same amount of interest to the original amount for each year that we accrue interest. We may generalise this calculation by using letters to represent the numbers involved:
  - Let $P$ represent the amount in investment and $n$ the number of years for a rate of interest denoted by $r$.
  - Let $A$ represent the total at the end of the year.
  - Then the interest earned $P \times \frac{r}{100} \times n = \frac{P \cdot r \cdot n}{100}$.
  - And the total at the end of the year is given by $A = P + \frac{P \cdot r \cdot n}{100}$.
  - If we convert the interest rate to decimal form, we may use $\frac{r}{100} = i$ to write the formula as $A = P + P \cdot i \cdot n$.
  - Buying goods on a hire purchase (HP) agreement is a financial context where simple interest rates are used.

- **When doing calculations with compound interest,** the interest is calculated on the accrued amount for each previous year, and then added to that accrued amount.
  - For now, it is sufficient just to know the formula for compound interest and how to apply it.
  - We use the same letters to represent numbers: $P, n, r, A$.
  - The formula for compound interest is: $A = P \left(1 + \frac{r}{100}\right)^n$.
  - Investments and loans are often calculated on the basis of compound interest.
  - For simple interest: $A = P + P \cdot i \cdot n$.
For compound interest:

\[ A = P \left(1 + \frac{r}{100}\right)^n \]

\[ = R2\ 000 \left(1 + \frac{15}{100}\right)^6 \]

\[ = R4\ 626,121531 \]

\[ = R4\ 626,12 \text{ (rounded to two decimal places)} \]

**Activity 1.13 Exchange rates**

The value of the money of other countries differs from ours. For example, for July 2012, the currency (money) introduced in Europe (called the euro) had a value of 10,25 times that of our South African Rand. We say that the exchange rate rand/euro is 10,25. It means you can get 1 euro for R10,25.

The exchange rates for the dollar $ (America) and pound £ (England) are given below:

<table>
<thead>
<tr>
<th>Currencies</th>
<th>rand/dollar</th>
<th>rand/pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchange rates</td>
<td>8,46</td>
<td>13,14</td>
</tr>
</tbody>
</table>

1. If an English tourist visits South Africa and exchanges her £2 000 for rands, how many rands will she get?

2. How many rands will an American tourist get for his $5 000?

3. If you visited England and exchanged your R6 000 for pounds, how many pounds will you get?

**Key ideas**

- If you have $100, and the R/$ exchange rate is 8,46, then your $100 is worth \( R8,46/\$ \times \$100 = R846 \).
- If on the other hand you have R100, then this will convert to \( \frac{R100}{R8,46} = \$11,82 \).
Activity 1.14 Accounts and VAT

This is Jane Racoon’s cellphone invoice for March 2012.

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
<th>Tax%</th>
<th>Tax</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airtime</td>
<td>13,74</td>
<td>14.00</td>
<td>1.92</td>
<td>15.66</td>
</tr>
<tr>
<td>Subscription</td>
<td>245,61</td>
<td>14.00</td>
<td>34.39</td>
<td>280.00</td>
</tr>
<tr>
<td>TELCEL insurance</td>
<td>42,54</td>
<td>14.00</td>
<td>5.96</td>
<td>48.50</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>301.89</td>
<td>14.00</td>
<td>42.27</td>
<td>344.16</td>
</tr>
</tbody>
</table>

1. Check the second line (subscription amount) of the invoice. If the cost is R245.61, will the VAT of 14% work out to R34.39? Show your calculation.

2. The VAT on airtime, subscription fees and insurance fees are worked out separately and then added to give R42.27. What happens if you work out the VAT on the total of R301.89? Will you get the same VAT of R42.27? Check this with a calculation and explain your answer.

Key ideas

- VAT is the 14% tax on certain items purchased. A music CD at R70 will have 14% VAT added to it and then carry a total price of VAT of \( \frac{14}{100} \times 70 = R9.80 \) PLUS the R70; i.e. total payment due will be R79.80.
- The separate calculations of VAT on each of the items detailed (and then totalled) is the same as the VAT on the total.

Activity 1.15 Profit, loss and discount

A storekeeper buys a pair of sunglasses for R150 from his suppliers.

1. How much must he sell the sunglasses for if he wants to make a profit of 200%?

2. If he attaches a selling price of R200 to the sunglasses, what is his percentage profit?

3. If the sunglasses priced at R200 has a 20% discount, what will the customer be required to pay?

4. If after three months the sunglasses remain unsold and the storekeeper reduces the price to R120, what will the percentage loss on such a sale be?
Key ideas

- These calculations are all calculations of percentage; for example, a selling price of R200 means that R50 of profit is made on the cost price of R150. The percentage profit is therefore calculated as $\frac{50}{150} \times 100$.
- Similarly, if the loss is from R150 to R120, then this amounts to R30 loss. Now calculate the percentage that R30 is of the cost, of R150.

Activity 1.16  Budgets

Sam wants to save up to travel home for the holidays. The table shows the budget Sam prepares of his weekly expenses:

<table>
<thead>
<tr>
<th>Costs</th>
<th>Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly salary</td>
<td>R1 150</td>
</tr>
<tr>
<td>Rent</td>
<td>R385</td>
</tr>
<tr>
<td>Food</td>
<td>R420</td>
</tr>
<tr>
<td>Cellphone</td>
<td>R65</td>
</tr>
<tr>
<td>Electricity</td>
<td>R40</td>
</tr>
<tr>
<td>Transport</td>
<td>R130</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td></td>
</tr>
</tbody>
</table>

1. a) Calculate Sam’s total weekly expenses.
   b) How much does Sam manage to save each week?

2. It will cost Sam R880 to travel home for the holidays. He will need a further R3 000 for presents for his family and R2 500 to spend while he is at home.
   a) How many weeks will it take for Sam to save up for his holiday?
   b) Will Sam be able to afford to take a three week holiday each year? Explain your answer.
   c) How much does Sam need to save each week in order to afford his holiday? Round off your answer to the nearest rand.
   d) How much extra does Sam need to save each month?
   e) What suggestions would you give to Sam about how he could save more money?
Key ideas

- A budget is a plan that shows what the expected income and expenses are for a period of time in the future.
- Money that someone earns or receives is called income and money that they spend is called expenditure.
- A budget helps you to plan how you will spend your money. You should always try to have enough money left over to save.
- Financial advisors tell you that you should save at least 5% of what you earn every month.

Activity 1.17  Rentals

The table that follows shows rental rates (prices) for the car rental company Superior Car Rental.

<table>
<thead>
<tr>
<th>Make of vehicle</th>
<th>Daily rates (R)</th>
<th>Long-distance rates (R)</th>
<th>Insurance per day (R)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Per day</td>
<td>Per km</td>
<td>Per day for 5 days or more, plus 200 km free per day</td>
</tr>
<tr>
<td>Sippalite</td>
<td>83</td>
<td>1,40</td>
<td>214</td>
</tr>
<tr>
<td>Tota 1.3</td>
<td>149</td>
<td>1,67</td>
<td>287</td>
</tr>
<tr>
<td>WV Pablo</td>
<td>208</td>
<td>2,19</td>
<td>378</td>
</tr>
<tr>
<td>Setta 2.0</td>
<td>369</td>
<td>4,12</td>
<td>681</td>
</tr>
</tbody>
</table>

*A waiver is an amount you pay in advance as insurance.

1. You hire a Tota 1.3 for a day to travel 50 km. The rental cost is R339.50. Explain how this total amount was calculated. Show all your calculations.

2. A client wants to hire a Tota 1.3 to make a trip around some parts of South Africa. She wants to cover distances of about 250 km per day and travel for a five-day period from Monday to Friday. Explain to her using a calculation why the ‘long-distance rates’ will work out much cheaper for this trip. To do this, you also have to show what it will cost her if she were to take the daily-rate option.

Key ideas

- The longer the rental period and distance covered, the higher the cost to the customer. This causes companies to make up different rental offers or packages.
• Rates are the prices given per km or per day. In mathematics, we can think of rate as the comparison of two quantities, such as km covered and cost. It makes more sense to say the rate is ‘R1,09 for 1 km’ than ‘R8,72 for 8 km’. Why?

**Exercise 1.9 Working with money**

1. Use a formula to calculate the value of an investment of R3 500 which accrues compound interest at 11,9% p.a. over five years.

2. Sam enters into a HP agreement for a microwave oven, which sells at R8 999 if bought cash. In terms of the agreement he is to pay 15% deposit, after which he will pay off the balance in equal monthly installments over three years at a simple interest rate of 24% p.a. Included in the monthly instalments is insurance of R17 per month. Calculate Sam’s monthly instalment and the amount he would have saved had he been able to pay for the appliance in cash. Round off your answer to two decimal places.

3. R4 200 is invested for four years and grows to R6 972. What is the simple interest rate required to achieve this growth?

4. How long would it take for an investment of R4 000 to grow to the value of R6 688 at a simple interest rate of 9,6% p.a.?

5. An artist sells a painting for R2 000 inclusive of VAT. How much money will the artist actually receive and how much money will go towards VAT?

6. Grace buys a pair of shoes on sale for R120. The original price of the shoes was R160. What percentage discount did she receive on the purchase?

7. Thandi needs R3 000 to complete a course at a university. Her aunt lends her the amount, but says that she will charge her 10% simple interest every year until the loan is repaid. How much will Thandi owe if she has not paid her aunt back any money in four years’ time?

8. Jake buys a radio on hire purchase. He pays back R45 each month for 18 months. What is the total amount that Jake paid for the radio? If the cash price of the radio was R600, what percentage extra did Jake pay?

9. John’s cousin in America buys him a pair of shoes for $99. If the rand/dollar exchange rate is 6,4, how many rands will it cost to pay her back?

10. John buys his American cousin jewellery worth R450. How many dollars will it cost his cousin to pay him back?
Summary

The first ten natural numbers = \{0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10\}.

- We can count beyond ten and never reach a ‘last number’ where our counting comes to an end. For this reason we represent the natural numbers as \( \mathbb{N} = \{0, 1, 2, 3, \ldots\} \), or simply just as \( \mathbb{N} \).
- We can perform the basic operations (+, −, ×, ÷) on the natural numbers.
- Extensions of the set of natural numbers (\( \mathbb{N} \)) may be summarised as follows:

<table>
<thead>
<tr>
<th>Extension to include...</th>
<th>Name of set formed</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negatives</td>
<td>Integers</td>
<td>( \mathbb{Z} )</td>
</tr>
<tr>
<td>Fractions</td>
<td>Rational numbers</td>
<td>( \mathbb{Q} )</td>
</tr>
</tbody>
</table>

- All the number systems above collectively form the real number system (denoted by the capital letter \( \mathbb{R} \)).
- The real number system may therefore be represented as:

\[
\begin{align*}
\text{REAL} & \subseteq \mathbb{R} \\
\text{Rational} & \subseteq \mathbb{Q} \\
\text{Integers} & \subseteq \mathbb{Z} \\
\text{Natural} & \subseteq \mathbb{N} \\
\text{Irrational} & \subseteq \mathbb{Q}'
\end{align*}
\]

- Every positive integer apart from 1, has a unique set of prime number factors.
- Prime numbers are positive integers (excluding 1) which have only two factors: themselves and 1.
- Numbers which are not prime numbers are called composite numbers: they have more than two factors.
- A ratio is a comparison of two or more quantities of the same kind, such as the volumes of two liquids that may be mixed together.
- A rate is a comparison of two different quantities. Speed is an example of a rate, which compares the distance travelled to the time taken to travel the distance.
- Two quantities are said to be in direct proportion if the ratio between corresponding values is constant.
- Two quantities are in inverse proportion if the product between corresponding values is constant. For simple interest the total at the end of the year is given by \( A = P + \frac{P \cdot r \cdot n}{100} \) or \( A = P + P \cdot i \cdot n \) where \( i = \frac{r}{100} \).
- For compound interest the formula is \( A = P\left(1 + \frac{r}{100}\right)^n \).
Check what you know

1. All of the following numbers are rational numbers:

{1, 3, −2.005, −16, 0, 5, 2 1/4, 7, 4,625, 8 3/4, 10, −33/45, 6, −1,000, −9}

a) What is meant by a rational number?
b) Explain why \(2\frac{1}{4}\) is a rational number.
c) Show that 5 is a rational number.
d) The decimal fraction 0.625 can be written as \(\frac{5}{8}\). Check whether this is correct and then explain why 4,625 is a rational number.
e) From the diagram above, write down the numbers that are rational numbers but not integers.
f) Write down only the integers.
g) Write down the natural numbers.
h) Write down an integer that is not a natural number.

Round off all non-integer answers to two decimal places.

A. The real number system

2. Choose the option(s) below which is/are correct. If an option is not correct, give a short mathematical reason for your answer:

a) The factors of 42 = \{1; 2; 7; 42\}.
b) The set of prime numbers less than 20 = \{1; 2; 3; 5; 7; 9; 11; 13; 17; 19\}
c) The answer of 100 ÷ 3 is a rational number.

3. Write the following numbers in ascending order with brackets:

a) The natural numbers less than 15
b) The prime numbers less than 15
c) The natural numbers between 20 and 50 that are multiples of 3
d) The prime numbers between 20 and 50

e) Even numbers which are also prime numbers

4. Calculate without using a calculator. State the smallest number system to which your answer belongs (i.e. whether it is natural, integer, rational or irrational), as well as whether the number is odd or even, prime or composite:

a) The difference between the first and last prime numbers which are less than 15

b) The sum of the first three prime numbers

c) The product of the greatest two prime numbers less than 20

d) The quotient of \((3 \times 8)\) and \((4 \times 12)\)

e) The LCM of 210 and 84 divided by 48.

B. HCF and LCM

5. Find the HCF and LCM of the following pairs or triples. Do the first two without using a calculator. You may use a calculator for c), d) and e).

a) 36 and 72       b) 225 and 90       c) 3 234 and 4 851

d) 30, 45 and 165   e) 408, 264 and 312

C. Ratio, rate and proportion

6. Express each ratio in its simplest form:

a) 3 : 15       b) 24 : 42       c) 25 : 45

d) 42 : 28 : 133  e) 52 : 91 : 39

7. Calculate the following. You may use a calculator.

a) i) A plane travels at an average speed of 513 km/h. How far does the plane travel in half an hour?

ii) Another plane travels at 850 km/h. How long would it take to travel from Cape Town to Johannesburg if the route taken is 1 263 km?

Give the answer in hours and minutes, rounded to the nearest minute.

b) Joe walked to his school from his home in 37 minutes. If his school is 3 km away from his home, calculate:

i) Joe’s average speed in metres per second (m/s)

ii) How far Joe would walk in an hour if he maintained the same average speed.
c) Mary can write a single page of a creative essay in 12 minutes. Martin can write a page of the same type of essay in 8 minutes.
   i) Which writer is the faster of the two?
   ii) How soon after the slower writer has started writing should the faster writer start if they want to finish a five-page essay at the same time?

d) If the cost of electricity is 107 c/kWh, calculate the cost of burning a 60 W light bulb for 40 minutes.
   (Note: kW = 1 kilowatt = 1 000 watts = 1 000 W).

e) The compound interest rate offered in an investment account is 12,3% p.a. for a minimum investment of R5 000. Calculate the minimum interest an investor could expect to earn from this account over three years.

8. Which of the following tables of values represent \(x\) directly proportional to \(y\), \(x\) inversely proportional to \(y\), or neither? If the relation is directly or inversely proportional, give the equation which describes it.

   a) \[
   \begin{array}{c|c}
   x & 2 & 3 & 4 & 5 \\
   \hline
   y & 14 & 21 & 28 & 35
   \end{array}
   \]

   b) \[
   \begin{array}{c|c|c|c}
   x & 1 & 2 & 3 \\
   \hline
   y & 2 & 1 & \frac{2}{3} \\
   \end{array}
   \]

   c) \[
   \begin{array}{c|c|c|c|c}
   x & 9 & 12 & 18 & 36 \\
   \hline
   y & 4 & 3 & 2 & 1
   \end{array}
   \]

   d) \[
   \begin{array}{c|c|c|c|c}
   x & -2 & -1 & 0 & 1 \\
   \hline
   y & -2 & -1 & 0 & 1
   \end{array}
   \]

   e) \[
   \begin{array}{c|c|c|c}
   x & 1 & 2 & 3 \\
   \hline
   y & 4 & 2 & 1
   \end{array}
   \]

   f) \[
   \begin{array}{c|c|c|c|c|c|c|c|c}
   x & 15 & 17 & 19 & 21 \\
   \hline
   y & -30 & -34 & -38 & -42
   \end{array}
   \]

D. Financial contexts

Profit, loss, discount, VAT

9. A pair of sports shoes is advertised on sale for 15% less than the normal price. Calculate the new selling price if the original price is R450. What is the amount of tax (VAT) that must be paid for each pair of shoes sold?

10. A cellphone company sells airtime for a pay-as-you-go top-up such that for a R50 voucher, you get an extra R10 free. Calculate the discount that this represents.

11. The factory price for Jake's Denim jeans (JD jeans) is R150. A retail store sells them for R400. What is the markup on a pair of JD jeans? Calculate the profit made on a consignment of 500 pairs of JD jeans.
12. Risky Deals is an investment company that specialises in high-risk investments. They invested R50 000 in a securities exchange in January 2008. By December 2009 they had lost 33% of their investment. Calculate the value of their investment in December 2009.

Simple interest, compound interest and HP

13. Patsy must make a decision between two investment options. She can choose to invest R20 000 in Growth Point Bank, which offers 12% compound interest p.a. or she can invest with Enterprise Bank, which offers a simple interest rate of 15% p.a. Which option gives the better return if she wants to invest for five years?

14. Penyo wants to buy a flat-screen TV, worth R12 000. The retailer offers a hire purchase plan for the flat screen, which includes the following details:
   - A deposit of 10% of the purchase price must be paid.
   - The balance must be paid over three years and will attract simple interest at a rate of 27% p.a.
   - Insurance on the goods is compulsory and is calculated as R15 per month in addition to the repayment instalments.
   a) Calculate the total monthly instalment if equal amounts are to be paid per month.
   b) Calculate the total amount that Penyo would have spent on the HP contract after his last instalment is paid.
   c) How much could he have saved had he bought the TV for R12 000?

15. How long would it take an investment of R2 300 to grow to at least R3 800 at a simple interest rate of 11,01% p.a.? Give the answer in a whole number of years.

16. If R8 000 grows in the ratio 5 : 2 over four years, calculate the equivalent simple interest rate per annum that would yield the same growth. In which context would the interest rate in your answer most likely be found – in an investment context or in the context of a loan, such as a hire purchase agreement?

Exchange rates, commissions and rentals

17. Thembi imports exotic perfumes from various countries and then sells them via her website. She also rents out office space at a rate of R200/m². The office space covers 15 m².
a) If the cost of a bottle of French perfume is 20 EUR, the cost of an Indian perfume 100 INR and the cost of a Turkish perfume 50 TRY, calculate the total cost (in ZAR) of importing a mixed consignment of 20 French, 20 Indian and 20 Turkish perfumes.

<table>
<thead>
<tr>
<th>One rand (ZAR) equals X currency</th>
<th>One currency equals X rand (ZAR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0,121904 US dollar (USD)</td>
<td>8,20 ZA rands</td>
</tr>
<tr>
<td>0,0785901 pound sterling (GBP)</td>
<td>12,72 ZA rands</td>
</tr>
<tr>
<td>0,0994597 euro (EUR)</td>
<td>10,05 ZA rands</td>
</tr>
<tr>
<td>6,75288 indian rupees (INR)</td>
<td>0,15 ZA rands</td>
</tr>
<tr>
<td>0,22157 new Turkish lira (TRY)</td>
<td>4,51 ZA rands</td>
</tr>
</tbody>
</table>

b) If she can sell a maximum of 15 consignments per month, calculate the profit that she is able to take as earnings if her markup is 15% on all perfumes. [Note: Assume that the only costs are those given in the question.]

c) Do you think that this is a sustainable business? Name one thing she could do in order to cut her costs of running the business.

18. Comfy Cars, a car-hire company, has two special 10-day offers for the mid-year holidays.

**Winter Wonder offer**
Any class A vehicle at R149 per day and R2,67/km. There is a flat charge of R500 for insurance for up to ten days.

**Mid-year Special offer**
Any class A vehicle at R287 per day and R1,67/km with a daily charge of R80 insurance for up to ten days.  
*plus – plus – plus*  
100 free kilometres per day!

a) If you went on a four-day holiday and travelled 600 km, which of the offers would be cheaper? Show your calculations.

b) If you went on a nine-day holiday and travelled an average of 200 km each day, which offer would you choose? Show your calculations.

c) Which offer would you choose if you needed to travel 1 500 km over two days? Show your calculations.
In this unit you will:

- perform calculations with integers in which we will revise:
  - calculations involving all four operations with integers
  - calculations involving all four operations with numbers that involve the squares, cubes, square roots and cube roots of integers
- revise properties of integers such as:
  - commutative, associative and distributive laws
  - additive and multiplicative inverses for integers
- solve problems in contexts involving multiple operations with integers.

Getting started

The questions and tasks that follow will give you some practice and revision of your work on integers in grades 7 and 8.

1. Copy the number line below and then complete the tasks that follow:

   
   
   
   
   
   
   
   
   
   
   

   Fill in the numbers that are missing on the number line, based on the following information:
   - the integer is –50
   - the integer is –20
   - the distance between 50 and the integer, is 60
   - the sum of the smallest integer shown on the number line and the biggest integer on the number line, is 10.

2. Compare the following integers by filling in <, =, >.

   a) –1 999 □ –2 000
   b) 10 – 22 + 33 □ 111 – 122 + 133
   c) –19 999 □ (–10 000) – 10 000

3. Refer to the number grid alongside:

   a) Check the sum of each row and each column.
   b) Find the sum of the rows.
   c) Find the sum of the columns.
   d) How do the sum of rows and sum of columns compare?
Key ideas

- Recall that integers are the set of negative and positive natural numbers and zero.
- The integer 50 is greater than 40, but −40 is greater than −50; on a number line the −50 is further left than the −40.
- Note too that −50 + 10 = −40; so −50 < −40.

Worked example

Work out the value of the following expressions:
1. \(25(3 − (−7)^2) + (5 − (−2)^2)^3\)
2. \(2(\sqrt[3]{−8} + \sqrt{64}) + (−2)^7\)
3. \((3 − 5)(5 − 3)\)

Solution
1. \(25(3 − (−7)^2) + (5 − (−2)^2)^3 = 25(3 − 49) + (5 − 4)^3 = 25(−46) + 1^3 = −1 149\)
2. \(2(\sqrt[3]{−8} + \sqrt{64}) + (−2)^7 = 2(−2 + 8) + (−128)\)
   \[= 2(6) − 128\]
   \[= 12 − 128\]
   \[= −116\]
3. \((3 − 5)(5 − 3) = (−2)(2) = −4\)

Exercise 2.1 Calculations with integers

1. Some numbers are missing on the number line that follows:

   A B C D
   −10 −8 −7 −6 −5 −4 −2 −1 0 1 2 3 4 5 7 8 10

   Which numbers are missing at A, B, C and D?

2. Fill in the missing integers on the number chain given below:

   [Diagram with numbers 35, 50, M, 4, (−47), K]

   \[\text{3. Calculate:}\]
   a) \(−\sqrt{64} − \frac{3}{\sqrt{125}}\)
   b) \(12 \times (5 − 10) + (−4)^2\)

   4. Work out \(\sqrt{225} + \frac{3}{\sqrt{512}}\).

   5. Check whether the following statements are true or false:
   a) \(−5^4 + 3^5 = −382; \text{ by } −5^4 \text{ we mean } −(5^4)\)
   b) \(−3^3 \times (−2)^5 = 864\)
Activity 2.1  Revising operations with integers

In grades 7 and 8 you learnt about adding, subtracting, multiplying and dividing integers. Let’s look at some of these rules for operations involving integers:

Work out the numerical value of the following number expression:

$$-2[3 - (-5)] + [9 - (-11)] ÷ (-5)$$

To work out the numerical value of this expression, we use the following convention:

- first perform operations inside brackets
- then, multiplication
- then division
- then addition
- then subtraction.

Here we follow the sequence or order of operations: First brackets, then multiplication, then division and then addition:

$$-2[3 - (-5)] + [9 - (-11)] ÷ (-5)$$

$$= -2[8] + [20] ÷ (-5)$$

$$= -16 + (-4)$$

$$= -20$$

1. Now try this method to calculate the numerical value of the expression:

$$2 × (-23 + 13) ÷ [(-15) + 10]$$

2. Convert the temperature of –39 °C to temperature readings in °F (Fahrenheit) using the formula, where °C stands for the temperature in degrees Celsius.

$$°F = (18 × °C + 320) ÷ 10$$

Key ideas

- The rules given in the activity are often referred to as ‘order of operations’, in which we agree on the order in which to perform operations. For example, we would first work out the expressions in brackets, then calculate the ‘multiplication’ parts, etc.
- You are already very familiar with addition and subtraction of integers. Here is a reminder of the rules for multiplication and division:

<table>
<thead>
<tr>
<th>+ integer × + integer = + integer</th>
<th>+ integer ÷ + integer = + number</th>
</tr>
</thead>
<tbody>
<tr>
<td>− integer × − integer = + integer</td>
<td>− integer ÷ − integer = + number</td>
</tr>
<tr>
<td>+ integer × − integer = − integer</td>
<td>+ integer ÷ − integer = − number</td>
</tr>
<tr>
<td>− integer × + integer = − integer</td>
<td>− integer ÷ + integer = − number</td>
</tr>
</tbody>
</table>

For multiplication  For division
Activity 2.2  Properties of integers

Here are some reminders of familiar properties of integers. Read through the properties and then answer the questions that follow:

1. The integers are **closed under addition** and **multiplication**. What does this mean?
2. The additive identity for integers is 0; explain this and give one example.
3. What is the multiplicative identity for the set of integers?
4. The inverse of any integer is an integer that yields 0 when the two are added, e.g. the inverse of –3 is + 3 because –3 + 3 = 0.
   a) What is the additive inverse of 10 and –45?
   b) If \( m \) and \( n \) are integers and \( m + n = 0 \), what can you say about the relationship between \( n \) and \( m \)?
5. Integers are commutative with respect to addition and multiplication. What does this mean? Use the number 7 and –3 to explain what you mean.

The **distributive rule** (for integers) is shown below:

The distributive rule:

\[
 a \times (b + c) = a \times b + a \times c 
\]

Check that \(-2 \times (3 - 4) = -2 \times 3 + (-2) \times (-4)\).

Key ideas

- For addition, properties are:
  - Integers are closed under addition; addition is associative and commutative.
  - The integers have 0 as identity for addition.
  - Each integer has an inverse.
- For multiplication, properties are:
  - Integers are closed under multiplication; multiplication is associative and commutative.
  - The multiplicative identity is 1.
  - Each integer does NOT have an inverse which is an integer where multiplication is the operation, for example, the inverse of 2 should be \( \frac{1}{2} \) because \( 2 \times \frac{1}{2} = 1 \); but \( \frac{1}{2} \) is not an integer (it is a fraction, or rational number).
Exercise 2.2  Using the properties of integers

1. Substitute $x$ and $y$ with the given values in order to work out the numerical values of the given expressions:
   a) $-4x^2 + 3(x + y)$, if $x = -8$ and $y = 2$
   b) $\sqrt[3]{x} + \sqrt[6]{y}$, if $x = -8$ and $y = 2$

2. Find the value of $x$ in each of the following equations:
   a) $3x - 3 = -15$
   b) $-5(x + 6) = 65$; the $x$-value is an integer between $-20$ and $20$.
   c) $x^2 - 16 = 0$; there are two possible values of $x$ that will work.
   d) $x(x - 10) = 0$; there are two possible $x$-values that will work.

3. When finding the sum of $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$, we may proceed as follows: $(1 + 9) + (2 + 8) + (3 + 7) + (4 + 6) + 10 + 5$
   a) Which property of integers allows us to rearrange the numbers this way?
   b) Which property of integers allows us to write $1 - 2 + 3 - 4 + 5 - 6 + 7 - 8 + 9 - 10$ as $(1 - 2) + (3 - 4) + (5 - 6) + (7 - 8) + (9 - 10)$?
   c) What is the sum of the integers in each case, a) and b)?

Activity 2.3  Algebra with integers

1. If $a$ and $b$ are integers, then we know that $ab = ba$. Which property of integers makes this statement true?

2. By the distributive rule, we have that $-2(a + b)a = (-2a - 2b)a$
   \[= -2aa - 2ba\]
   \[= -2a^2 - 2ab\]

   Explain how we proceed from the expression $-2(a + b)a$ to the step that follows from it.

3. Solve for $x$ in the equation $-3(x - 5) = 30$. A procedure is shown:
   -3($x - 5$) = 30 \hspace{1cm} \text{line 1}
   so $-3x - 3(-5) = 30 \hspace{1cm} \text{line 2}$
   therefore $-3x + 15 = 30 \hspace{1cm} \text{line 3}$
   now $-3x + 15 - 15 = 30 - 15 \hspace{1cm} \text{line 4}$
   so $-3x + 0 = 15 \hspace{1cm} \text{line 5}$
   and so, $-3x = 15 \hspace{1cm} \text{line 6}$
   but we know $-3x = 15 = -3 \times -5 \hspace{1cm} \text{line 7}$
   so $x$ must be $-5$. \hspace{1cm} \text{line 8}
Write down the procedure for solving the equation and give reasons or explanations for each line 2–8; for example, to go from line 1 to 2 we use the distributive rule.

4. Simplify the following expression: \(5(p + q^2) - 2(2p - 3q^2)\). Explain the various steps in your simplification.

**Key ideas**

- For integers \(a\) and \(b\), we have that \(ab = ba\) since integers are commutative with respect to multiplication.
- Similarly the other rules apply – in Question 2 we use the distributive rule.
- When solving equations, as in Question 3, you will notice the use of the distributive rule in Line 2, the use of additive inverses and the identity in lines 4 and 5.
- To simplify the expression using the rules of operation on integers and properties of integers we have:

\[
5(p + q^2) - 2(2p - 3q^2) \\
= 5p + 5q^2 - 4p + 6q^2 \\ 
= 5p - 4p + 5q^2 + 6q^2 \\
= (5p - 4p) + (5q^2 + 6q^2) \\
= p + 11q^2
\]

- **Note:** we treat expressions such as \(ab, a + b, a ÷ b, a - b\), where \(a\) and \(b\) are integers, in the same way as if we are working with actual numerical-valued integers; so for example, \(2a + 3b - 6a - 10b = 2a - 6a + 3b - 10b = -4a - 7b\).

**Worked example**

1. Solve the equation \(-2(x + 1) = -4\), giving reasons for all your steps.

**SOLUTION**

\[-2(x + 1) = -4 \quad \text{[the value on either side of the equal sign is the same]}\]

\[\therefore -2x + -2.1 = -4 \quad \text{[applying distributive rule to } 2(x + 1)]\]

\[\therefore -2x - 2 = -4 \quad \text{[multiplying out, } -2.1 = -2\]}

\[\therefore -2x - 2 + 2 = -4 + 2 \quad \text{[adding the additive inverse of } -2 \text{ on either side of the equation, i.e. add 2]}\]

\[\therefore -2x + 0 = -2 \quad \text{[calculating } -2 + 2 \text{ as 0 and } -4 + 2 \text{ as } -2]}\]

\[\therefore -2x = -2 \quad \text{[-}2x + 0 \text{ is } -2x \text{ (as 0 is identity for addition)]}\]

\[\therefore -2x = -2 \times 1 \quad \text{[rewriting } -2 \text{ as } -2 \times 1, \text{ which we can do as 1 is identity element]}\]

so \(x = 1\) \[\text{[comparing left- and right-hand side of the equation we conclude that } x = 1\]

40 Term 1 • Unit 2
2. Simplify the following expression: \(x(x + 3) + 2(x + 3)\)

**SOLUTION**

\[
x(x + 3) + 2(x + 3) = x^2 + 3x + 2x + 6 \quad \text{[by the distributive rule]}
\]

\[
x^2 + 5x + 6 \quad \text{[adding like terms]}
\]

3. Simplify the expression \(35s(1 - s^2) ÷ (-5)\).

**SOLUTION**

\[
35s(1 - s^2) ÷ (-5)
\]

\[
= \frac{35s(1 - s^2)}{-5}
\]

\[
= -7s(1 - s^2)
\]

\[
= -7s + 7s^3
\]

### Exercise 2.3  Algebra and integers

1. Simplify: \(3(a - (-b)^2) + 2(a - (-b)^2)\)

2. Simplify: \(x(\sqrt{a} - 8 + \sqrt{a^2}) + (-2)^7\)

3. Simplify: \(z(y - z) - y(z - y)\)

4. Solve the equation \(5x + 16 = 6\) and show all steps with explanations involving the properties and rules of operation involving integers.

5. Simplify the expression \(-4(5 - w) + 3(w - 5)\); say where you use any of the properties or rules of operations of integers.

6. The distributive and commutative properties of integers are used in the simplification that follows:

\[
7n(2y - 3p) - 2p(2y - 3p)
\]

\[
= 14ny - 21np - 4py + 6p^2
\]

\[
= 14ny - 4py - 21np + 6p^2
\]

a) State in which line which property is used.

b) If \(y = -1, n = 3\) and \(p = -2\), calculate the value of the expression.

7. For \(x \neq 0\), simplify the expression \(\frac{30x(x - 1)}{-9x^2}\) and then find the value of the expression if \(x = -10\).

8. What is the additive inverse of the following integers?

a) \(n\)

b) \(-m\)

c) \(p - q\)
Activity 2.4  Solving a problem involving integers and algebraic expressions

The height above sea level of a mountain peak is $5h$ metres and the height of the ocean floor $-2h$ metres as shown:

1. What will the expression $5h + (-2h)$ represent?
2. What will the expression $5h + 2h$ represent?
3. If the total distance from peak to ocean floor (bed) is 3 500 metres, how high is the peak above sea level? Show your workings.

Key ideas

- When we use 0-level as sea level, we can decide to have height above sea level as positive and depth below sea level as negative; thus $-2h$ means depth below sea level.
- While $-2h$ indicates distance below sea level, the distance from the seabed to the mountain peak is given by the difference $5h - (-2h) = 5h + 2h = 7h$.
- Thus the distance from seabed to the peak is $7h = 3500$ m, which is $7h = 7(500)$ m and therefore $h = 500$ m.
  The peak is therefore $5h = 5(500) = 2500$ m above sea level.

Exercise 2.4  Using integers to solve problems

1. Look at the table of temperatures in degrees Celsius (°C) and Kelvin (K):

<table>
<thead>
<tr>
<th>Temperature in Celsius (°C)</th>
<th>Temperature in Kelvin (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body temperature</td>
<td>37</td>
</tr>
<tr>
<td>Ice water</td>
<td>0</td>
</tr>
<tr>
<td>Boiling water</td>
<td>100</td>
</tr>
</tbody>
</table>
a) How do you work out the Celsius temperature if you know the Kelvin temperature?

b) If some temperature is measured in Kelvin and is given as \( x \) Kelvin, how will you write out the Celsius temperature?

c) How many degrees Celsius is 0 Kelvin?

d) Sometimes temperature is also measured in degrees Fahrenheit (°F). If a temperature is \( t \) degrees Celsius, you can convert it to Fahrenheit by using the following calculation (rule or formula): \((1.8 \times t) + 32\).

i) What will 37 °C be in Fahrenheit?

ii) Work out the temperature in Celsius, if it is given as 212 °F.

2. If the square grid alongside has all rows, columns and diagonals adding up to the same number, it will be a magic square.

a) Find the values of \( a \), \( b \), \( c \) and \( d \) that will make the square a magic square.

b) Calculate the numerical value of the expression \(-3a + b + d\).

3. A new car of cash value R100 000, depreciates in value (loses value) over a period of five years; the depreciated value \( d \) can be calculated by the formula \( d = 100\ 000\left(1 - \frac{nr}{100}\right)\), where \( r \) is the percentage depreciation per year and \( n \) is the number of years (after the car has been bought). Calculate the value of the car after two years if the depreciation rate \( r \) is 20% per year.

**Summary**

- Integers are made up from the set of negative and positive natural numbers and zero.

- The rules given in the activity are often referred to as ‘order of operations’, in which we agree on the order in which to perform operations.

- You are already very familiar with addition and subtraction of integers. Here is a reminder of the rules for multiplication and division:

| + integer \( \times \) + integer | = | + integer | + integer \( \div \) + integer | = | + number 
| + integer \( \times \) - integer | = | + integer | - integer \( \div \) - integer | = | + number 
| - integer \( \times \) + integer | = | - integer | + integer \( \div \) - integer | = | - number 

For **multiplication**

For **division**
For addition, properties are:
• Integers are closed under addition; addition is associative and commutative.
• The integers have 0 as identity for addition.
• Each integer has an inverse.

For multiplication, properties are:
• Integers are closed under multiplication; multiplication is associative and commutative.
• The multiplicative identity is 1.
• Each integer does NOT have an inverse which is an integer where multiplication is the operation, for example, the inverse of 2 should be \( \frac{1}{2} \), because \( 2 \times \frac{1}{2} = 1 \); but \( \frac{1}{2} \) is not an integer (it is a fraction, i.e. rational number).

Check what you know

1. Calculate:
   a) \(-\sqrt{81} - \sqrt{-64}\)
   b) \(-5 \times (8 - 28) + (-3)^3\)
2. Work out \(\sqrt{25} + \frac{3}{1,000}\)
3. Check whether the following statements are true or false:
   a) \((-3)^4 + 5^3 = -44\)
   b) \(-6^3 \times (-2)^7 = 344\)
4. If the square grid alongside has all rows, columns and diagonals adding up to the same number, it will be a magic square.
   a) Copy the grid (magic square) and fill in the missing numbers that will make it a magic square.
   b) Calculate the sum of all the numbers in the square.
5. Use \(p = -7\) and \(q = 5\) to see whether \(p + p + q + 3 = 2p + q + 3\).
   Try other \(p\) and \(q\) number pairs to check if whether it remains true.
6. a) Check that \(5a + 3b = 8ab\) if \(a = 0\) and \(b = 0\).
   b) If \(a = -1\) and \(b = -1\), is \(5a + 3b\) still equal to \(8ab\)?
   c) What if \(a = 2\) and \(b = -3\)?
7. Rewrite \(-10x + c + 23x + 26c -15x - 10c\) as an expression with one term in \(x\) and one term in \(c\). State where you use commutative and associative properties.
8. Work out the value of the following expressions:
   a) \( x(3x - (-4)^2) + (2x - (-3)^2)x \)
   b) \( y(\sqrt[3]{-8x^3} + \sqrt{64x^2}) + xy(-3)^3 \)
   c) \( (x - y)(y - x) \) if \( x = 33 \) and \( y = -67 \)

9. The solution to the equation \(-5(2x - 1) = -52\) is given below. Most of the reasons are supplied; copy the solution and then fill in the missing reasons a)–d).

   **SOLUTION**
   
   \[-5(2x - 1) = -5^2 \quad [\text{the value on either side of the equal sign is the same}]\]
   \[\therefore -10x - 5(-1) = -25 \quad [\text{a) which rule/property is applied here?}]\]
   \[\therefore -10x + 5 = -25 \quad [\text{multiplying out, \(-5(-1) = +5\)}]\]
   \[\therefore -10x + 5 + (-5) = -25 + (-5) \quad [\text{b) what is the additive inverse added on both sides of the equation?}]\]
   \[\therefore -10x + 0 = -30 \quad [\text{c) how is this line obtained from the previous one?}]\]
   \[\therefore -10x = -30 \quad [-10x + 0 \text{ is } -10x \text{ (as 0 is the identity for addition)}]\]
   \[\therefore -10x = -10 \times 3 \quad [\text{d) explain the reasoning here}]\]
   so \( x = 3 \quad [\text{comparing left- and right-hand side of the equation we get that } x = 3]\)

10. Simplify the following expression: \( x^2(x - 1) - 2x(x - 1) + x - 1 \)
    In other words, use the distributive rule to multiply out the brackets and the commutative rule to add like terms.

11. An integer and its additive inverse have a sum of \___________.

12. The product of a non-zero integer and its additive inverse is always a) negative or b) positive?

13. The product of an integer and its additive inverse is \(-400\). What integer is it?

14. Copy the diagram and then fill in the missing expressions or values in the open circles:

   ![Diagram](image)

   Start at the circle containing 4 and then continue clockwise.
15. △ABC shown below is translated by 7 units horizontally to the left. What will the coordinates of the translated triangle be?

Second quadrant  
First quadrant

16. If a point \((x; y)\) is translated by 3 units left and then 4 units down, which one of the following will its coordinates be?

- a) \((x + 3; y + 4)\)
- b) \((x - 3; y - 4)\)
- c) \((x - 3; y + 4)\)
- d) \((x + 3; y - 4)\)

17. A stone is thrown from the top of a building; the rooftop is 80 metres above ground level.

The height \(h\) of the stone above the building after a time of \(t\) seconds is given by the formula \(h = 10t - 5t^2\).

- a) What is the height of the stone above the rooftop after 1 second?
  You can calculate this height by substituting \(t = 1\) into the formula.
- b) What is the height of the stone above the rooftop after 2 seconds?
  What does the answer mean?
- c) What can you say about the height of the stone after 3 seconds?
In this unit you will:

• perform calculations with fractions involving
  o all four operations with common fractions and mixed numbers
  o all four operations with numbers that involve squares, cubes, square roots
    and cube roots of common fractions
• also revise Grade 8 calculations involving fractions including:
  o convert mixed numbers to common fractions in order to perform
    calculations with them
  o use knowledge of multiples and factors to write fractions in the simplest
    form before or after calculations
  o use knowledge of equivalent fractions to add and subtract common fractions
  o use knowledge of reciprocal relationships to divide common fractions
  o use knowledge of decimal fractions, percentages, common fractions and
    mixed numbers to solve problems
• be introduced to the concepts outlined above in algebraic expressions and
  equations (new to Grade 9).

Getting started  Equivalent fractions

1. Three fractions $\frac{3}{8}$, $\frac{1}{2}$ and $\frac{3}{4}$ are added as shown below:

$$\frac{3}{8} + \frac{1}{2} + \frac{3}{4} = \frac{3}{8} + \frac{4}{8} + \frac{6}{8} = \frac{13}{8} = 1\frac{5}{8}$$

   Explain how this addition was done.

2. Add the fractions given below:
   a) $\frac{12}{8} + \frac{3}{4}$  b) $\frac{9}{6} + \frac{25}{10}$  c) $\frac{1}{2} + \frac{1}{3}$  d) $1\frac{1}{2} + \frac{2}{5}$

3. The difference between $\frac{3}{4}$ and $\frac{2}{3}$ is $\frac{1}{12}$. Show how you will calculate this
   difference to get the answer.

4. Discuss and compare methods for adding or subtracting fractions in class.
Key ideas

- Fractions that have the same denominators are easy to add; you just add the numerators: \( \frac{3}{8} + \frac{4}{8} + \frac{6}{8} = \frac{13}{8} \)

  If the denominators differ, you can use equivalent fractions to make the denominators the same:

  \[
  \begin{align*}
  \frac{1}{2} & \quad + \quad \frac{1}{3} & \quad + \quad \frac{1}{4} \\
  \frac{6}{12} & \quad + \quad \frac{4}{12} & \quad + \quad \frac{3}{12}
  \end{align*}
  \]

  equivalent fractions with same denominator, 12

- It is easy to compare fractions when their denominators are the same.

  From above \( \frac{1}{2} \) is bigger than \( \frac{1}{3} \); they differ by \( \frac{2}{12} \), because \( \frac{6}{12} - \frac{4}{12} = \frac{2}{12} \).

Activity 3.1  Division and multiplication

A fraction model (or fraction wall) for halves, thirds, quarters, sixths and twelfths is given:

<table>
<thead>
<tr>
<th>halves</th>
<th>thirds</th>
<th>quarters</th>
<th>sixths</th>
<th>twelfths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. How many of \( \frac{2}{12} \) in \( \frac{3}{6} \)? In other words, what is \( \frac{3}{6} \div \frac{2}{12} \)? These fractions could be shaded on the fraction model, as shown:

- \( \frac{3}{6} \) marks sixths
- \( \frac{2}{12} \) marks twelfths

2. Compare your answer above with \( \frac{3}{6} \times \frac{12}{2} \), which is \( \frac{36}{12} \) or just 3.

3. In the same way as in questions 1 and 2, work out \( \frac{1}{2} \div \frac{1}{6} \) and then compare your answer with \( \frac{1}{2} \times \frac{6}{1} \).
4. How does the answer to
   a) \( \frac{2}{3} \div \frac{1}{6} \) compare with \( \frac{2}{3} \times \frac{6}{1} \)?
   b) \( \frac{3}{4} \div \frac{3}{12} \) compare with \( \frac{3}{4} \times \frac{12}{3} \)?

5. You can work out that \( \frac{9}{8} \times \frac{2}{3} = \frac{18}{24} \), or \( \frac{3}{4} \) (the equivalent fraction).
   a) If you reverse the calculation, then \( \frac{3}{4} \div \frac{2}{3} \) must be \( \frac{9}{8} \). Do you agree?
   b) Compare the answer to \( \frac{3}{4} \div \frac{2}{3} \) to the answer of \( \frac{3}{4} \times \frac{3}{2} \).
   c) Explain what you notice about the division of fractions when multiplication is introduced. Does it help you to change the division of fractions into a multiplication of fractions calculation with the same answer?

**Key ideas**

- You can change the ‘division of fractions’ into a ‘multiplication of fractions’ calculation, without changing the answer; that is, \( \frac{a}{b} \div \frac{c}{d} \) becomes \( \frac{a}{b} \times \frac{d}{c} \).
- The fraction that you divide with must be inverted (‘turned upside down’) when you change division to multiplication.

**Exercise 3.1 Using fractions**

1. The money that covers a school’s expenses per year is shown in the pie chart alongside:
   a) What fraction of the total of the school’s income is obtained through donations?
   b) The white region shows the fraction of the money that comes in through school fees. What fraction is it of the total income?
   c) What fraction of the total is gained through fund-raising?
   d) If the fund-raising amount is R20 000, what will each of the other amounts, school fees and donations, be?

2. From the model of fractions you can see that the fractions \( \frac{3}{2}, \frac{6}{4}, \frac{12}{8} \) and \( \frac{24}{16} \) are equivalent fractions.
a) Why are they equivalent fractions?
b) Use the model to find two fractions equivalent to $\frac{3}{4}$.
c) Work out the top numbers (numerators) and bottom numbers (denominators) of the equivalent fractions without looking at the fraction model. Explain how you do it.
d) If the fractions $\frac{9}{12}$, $\frac{15}{18}$ and $\frac{\square}{32}$ are all equivalent to $\frac{3}{2}$, what must the missing numerators and denominators be? Show your calculations.
e) What can you say about the fractions $\frac{6}{8}$ and $\frac{16}{16}$?
f) Explain why the fractions $\frac{13}{8}$ and $\frac{11}{8}$ are equivalent (the same).
g) How many sixteenths will $1\frac{7}{16}$ make? That is, write $1\frac{7}{16}$ in the form $\frac{\square}{16}$.
h) Write $\frac{5}{4}$ in the form $1\frac{\square}{4}$; that is, ‘1 and how many quarters?’
i) Explain how you will add this: $\frac{1}{8} + \frac{\square}{8} + 1\frac{7}{8}$ to get an answer of $4\frac{3}{8}$.

3. Calculate the following fractions:
   a) $\frac{3}{4} + \frac{5}{8}$
   b) $\frac{4}{6} - 2\frac{1}{2}$
   c) $\frac{2}{3} - \frac{12}{15} + \frac{3}{5}$
   d) $2\frac{1}{2} \times 2\frac{1}{4}$
   e) $\frac{2}{3} \div \frac{4}{3}$

Activity 3.2  Choosing the lowest common denominator (LCM)

Here is a procedure for adding fractions using the LCM.

**Question:** Simplify: $\frac{1}{2} + \frac{3}{4} + \frac{2}{5}$

**Procedure**
First choose a common denominator into which all three denominators can divide.

Look at the following common denominators:
- 40 works because 2, 4 and 5 can divide into it
- 60 also works because 2, 4 and 5 can divide into it
- 20 is the smallest number that 2, 4 and 5 can divide into.

All of the common denominators will work and give a correct answer to the sum, but choosing the **lowest common denominator** makes the sum work out more easily.
SOLUTION
\[ \frac{1}{2} + \frac{3}{4} + \frac{2}{5} = \frac{10}{20} + \frac{15}{20} + \frac{8}{20} = \frac{10 + 15 + 8}{20} = \frac{33}{20} \]

1. Use this method to complete the given sum; that is, fill in the missing numbers.
\[ \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6 + \Box + \Box}{12} = \Box \]

2. Some of the common denominators that you could use when working out \( \frac{2}{3} + \frac{3}{5} \) are 30, 15 or 45. Work out the sum of the two fractions.

3. Work out \( \frac{5}{6} + \frac{3}{4} \).

Key ideas
The smallest or lowest, common (same) denominator LCD or LCM is used to make addition of fractions easier. This method is often used; use it if it suits you.

Worked example
A mixed number is commonly higher than 1. Mixed numbers are also called improper fractions when written as \( \frac{3}{2}, \frac{10}{5}, \) etc. Examples of mixed numbers are \( 1\frac{1}{2}, 3\frac{3}{3}, 5\frac{5}{4}, \) etc.
Calculate the following: \( \frac{7}{4} + 1\frac{1}{3} \)

SOLUTION
Convert the mixed number \( 1\frac{1}{3} \) into an improper fraction to allow for easier calculation. 1 can be written as \( \frac{3}{3} \) therefore \( 1\frac{1}{3} \) equals \( \frac{3}{3} + \frac{1}{3} \) (we write 1 with a common denominator of 3 because of the fraction \( \frac{1}{3} \) that is attached to it). Therefore \( \frac{3}{3} + \frac{1}{3} = \frac{4}{3} \).

Now we can write the above problem as: \( \frac{7}{4} + \frac{4}{3} \)
The LCD (lowest common denominator here is 4 \( \times \) 3 = 12).
Therefore \( \frac{7(3)}{4(3)} + \frac{4(4)}{3(4)} = \frac{21}{12} + \frac{16}{12} \).

Now we add the numerators and get one fraction.
\( \frac{37}{12} \) or it can be written as \( 3\frac{1}{12} \).

Worked example
When we simplify fractions, we change them into their simplest form.
Calculate the following: \( \frac{6}{3} + \frac{21}{7} - \frac{16}{4} + \frac{128}{256} \)
SOLUTION

Simplify all the fractions into their lowest denominators.

So we have:

\[ \frac{6 \div 3}{3 \div 3} = \frac{2}{1}, \quad \frac{21 \div 7}{7 \div 7} = \frac{3}{1}, \quad \frac{16 \div 4}{4 \div 4} = \frac{4}{1}, \quad \frac{128 \div 128}{256 \div 128} = \frac{1}{2} \]

The problem can now be written as: \( \frac{2}{1} + \frac{3}{1} - \frac{4}{1} + \frac{1}{2} \)

The LCD is 2 so we rewrite it as:

\[
\frac{2(2)}{1(2)} + \frac{3(2)}{1(2)} - \frac{4(2)}{1(2)} + \frac{1}{2} = \frac{4 + 6 - 8 + 1}{2} = \frac{3}{2}\]

as an improper fraction or as a mixed number \(1 \frac{1}{2}\)

Worked example

The fractions \(\frac{1}{2}, \frac{3}{6}, \frac{43}{86}, \frac{6}{12}\) all have the same value but their common form is \(\frac{1}{2}\).

Using this knowledge, solve the following: \(\frac{5}{25} - \frac{28}{56} + \frac{3}{24}\)

SOLUTION

The equivalent of \(\frac{5}{25}\) is \(\frac{1}{5}\); \(\frac{28}{56}\) is \(\frac{1}{2}\); \(\frac{3}{24}\) is \(\frac{1}{8}\).

Rewriting the problem with the common equivalent we have: \(1 \frac{5}{25} - \frac{28}{56} + \frac{3}{24}\)

Convert the mixed number \(1 \frac{5}{25}\) to an improper fraction then: \(\frac{6}{5} - \frac{1}{2} + \frac{1}{8}\)

The LCD is 40 as the denominators 5, 2 and 8 can be divided equally into 40. If we did not simplify it first, then we would have had to use a LCD of \(25 \times 56 \times 24 = 33600\).

Simplifying the problem now into one fraction using the LCD of 40 we have:

\[
\frac{6(8) - 1(20)}{5(8) - 2(20) + 1(5)} = \frac{48 - 20 + \frac{5}{40}}{40 - 40 + \frac{5}{40}} = \frac{33}{40}
\]

Worked example

Reciprocals are two numbers that when multiplied by each other give 1.

For \(\frac{1}{2} \times \frac{2}{1} = 1\), the reciprocal of \(\frac{1}{2}\) is \(\frac{2}{1}\).

When you divide by a number, it is the same as multiplying by its reciprocal.

Solve the following: \(\frac{1}{3} \times \frac{4}{7} \div \frac{24}{72}\)
Before we start calculating anything, we first have to simplify any fractions we can. \( \frac{24}{72} = \frac{1}{3} \), so we have: \( \frac{1}{3} \times \frac{4}{7} \div \frac{1}{3} \). Using the definition of the reciprocal, we can rewrite this problem as \( \frac{1}{3} \times \frac{4}{7} \times \frac{3}{1} \) (the reciprocal of \( \frac{1}{3} \) is \( \frac{3}{1} \) and note that we changed the \( \div \) to \( \times \)). Solving this we get \( \frac{4}{7} \).

### Exercise 3.2

Four operations with fractions

| 1. \( \frac{2}{3} + \frac{1}{4} \) | 2. \( \frac{2}{3} \times \frac{3}{4} \) | 3. \( \frac{2}{3} \div \frac{3}{4} \) |
| 4. \( \frac{2}{3} - \frac{3}{4} \) | 5. \( \frac{2}{5} + 1\frac{1}{2} - \frac{3}{10} \) | 6. \( -\frac{3}{2} - \frac{1}{6} \) |
| 7. \( \frac{14}{15} \times \frac{20}{21} \times \frac{3}{10} \) | 8. \( \frac{10}{27} \times \frac{9}{14} \div \frac{5}{3} \) | 9. \( \frac{4^2}{5} + \frac{2}{5}(-3)^2 \) |

### Activity 3.3

Percentage and fractions

The following percentages and decimals must be written as fractions and then presented in their simplest form as a fraction:

1. 73% of 250
2. 2.75

Try this task first and then look at the working out. Answer the questions that accompany them:

**SOLUTION**

1. 73% can be written as \( \frac{73}{100} \).
   
   Therefore \( \frac{73}{100} \) of 250 = \( \frac{73}{100} \times \frac{250}{1} \).
   
   Now divide by \( \frac{50}{50} \) to simplify: \( \frac{73}{2} \times \frac{5}{1} = \frac{365}{2} \).

   Explain what is done in lines 1, 2 and 3.

2. 2.75 can be written as \( \frac{275}{100} \). Simplifying we get:
   
   \( \frac{23}{4} \) and written as an improper fraction \( \frac{11}{4} \). Explain the procedure.

### Key ideas

- Remember that we write percentage in fraction form as parts of a hundred; so 10% means 10 parts of a hundred or \( \frac{10}{100} = \frac{1}{10} \).
- Each place to the right of the decimal comma decreases in tenths: so 0.234 means 0 units, 2 tenths, 3 hundredths and 4 thousandths: \( 0 + \frac{2}{10} + \frac{3}{100} + \frac{4}{1000} \).
Worked example
Write a number sentence to solve this:
A man buys an airtime voucher of R50,00 from Dovacom. As he is a valued customer of Dovacom for three years now, he gets an extra 15% value of the purchased voucher. How much airtime in monetary terms (Rands) did he get?

SOLUTION
To calculate 15% of R50,00 we use its fraction form:
\[
\frac{15}{100} \times R50,00 \text{ (which is the extra bit he gets) plus R50,00 (which he bought originally)}
\]
Solving we get: R7,50 + R50,00 = R57,50
Alternatively, since this man is getting an extra 15%, he will be getting 115% of the value of the voucher bought. Using this as our basis we can write the solution as: 115% of R50,00 or \(1,15 \times R50,00\) which translates to: \(\frac{115}{100} \times R50,00 = R57,50\)

Worked example
Write a number sentence to solve this:
Lucy went to the fruit store to buy peaches. She decided to buy a bag of peaches valued at R25,00. At the same time she cashes in her 30% off any purchase coupon at that store. How much will she pay for the peaches?
The value of the peaches is R25,00 and 30% of that can be written as:
\[
\frac{30}{100} \times R25,00 = R7,50
\]
Hence the total value of the discounted peaches is: R25,00 – R7,50 = R17,50
Alternatively we can do this: since Lucy is getting a 30% discount she will only be paying 70% of the value of the peaches (100% – 30% = 70%), so we can write the solution as: 70% of R25,00 or \(\frac{70}{100} \times R25,00 = R17,50\)

Activity 3.4 Squares and roots of fractions
Write down explanations for the procedures on the right:
1. \(\left(\frac{3}{4}\right)^2 = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}\)
2. \(\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}\)
3. \(\sqrt{\frac{-210}{27}} = \frac{\sqrt{-64}}{\sqrt{27}} = \frac{-4}{\sqrt{3}}\)
Key ideas

- You can use the following rules for squares and roots (where \(a\) and \(b\) are any real numbers, but \(b \neq 0\)):
  - \((\frac{a}{b})^2 = \frac{a^2}{b^2}\) and \((\frac{a}{b})^3 = \frac{a^3}{b^3}\)
  - \(\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}\) and \(3\sqrt[3]{\frac{a}{b}} = \frac{3\sqrt[3]{a}}{\sqrt[3]{b}}\)
  - You can generalise these rules: \((\frac{a}{b})^n = \frac{a^n}{b^n}\) and \(n\sqrt[3]{\frac{a}{b}} = \frac{n\sqrt[3]{a}}{\sqrt[3]{b}}\) where \(b \neq 0\).

Worked example

1. Calculate \(\left(\frac{4}{5}\right)^2\).

   **SOLUTION**
   
   This is the equivalent of saying \(\frac{4}{5} \times \frac{4}{5}\). Calculating it we get:
   
   \[
   \frac{4}{5} \times \frac{4}{5} = \left(\frac{4}{5}\right)^2
   \]
   
   \[
   = \frac{16}{25}
   \]

2. Calculate \(\sqrt{2\frac{14}{25}}\).

   **SOLUTION**
   
   The first thing we have to do here is to convert the fraction in the square root to an improper fraction, in this case \(2\frac{14}{25} = \frac{64}{25}\). Remember from Grade 8 that we can simplify it by working out the square root of the numerator and denominator separately:
   
   \[
   \sqrt{\frac{64}{25}} = \frac{8}{5}
   \]

Exercise 3.3

<table>
<thead>
<tr>
<th></th>
<th>Squares and roots of fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>(\sqrt{2\frac{1}{4}})</td>
</tr>
<tr>
<td>2.</td>
<td>(-3 \div \frac{1}{5})</td>
</tr>
<tr>
<td>3.</td>
<td>(\frac{3}{\sqrt{\frac{1}{8}}})</td>
</tr>
<tr>
<td>4.</td>
<td>(\frac{1}{2} + \frac{3}{1 - \frac{5}{6}})</td>
</tr>
<tr>
<td>5.</td>
<td>(2\frac{3}{7} \times 1\frac{1}{13})</td>
</tr>
<tr>
<td>6.</td>
<td>(\frac{2}{5} + 1\frac{1}{2} - \frac{3}{10})</td>
</tr>
<tr>
<td>7.</td>
<td>(\left(\frac{4}{5}\right)^2 + \frac{2}{5}(-3)^2)</td>
</tr>
<tr>
<td>8.</td>
<td>(\sqrt{\frac{1}{5} + \left(\frac{2}{5}\right)^2})</td>
</tr>
<tr>
<td>9.</td>
<td>20% of 1 500</td>
</tr>
<tr>
<td>10.</td>
<td>5% of 300</td>
</tr>
<tr>
<td>11.</td>
<td>(\frac{1}{4}) of 45%</td>
</tr>
</tbody>
</table>

12. Write the following fractions in ascending order (smallest to biggest):
   
   **a)** \(\frac{1}{2}, 5\frac{2}{6}, \frac{3}{3}, \frac{3}{4}\)  
   **b)** 0.4; \(\frac{14}{25}\); 37%; \(\frac{13}{20}\); \(\frac{3}{5}\)

13. Annie wants to bake small apple pies. She uses three-quarters of an apple per pie. How many pies can she make with 12 apples?
14. A high school has 980 learners. One quarter of these learners are in Grade 9. Two-fifths of the Grade 9s are girls. How many boys are in Grade 9?

15. There are 135 learners that play soccer at a school. Three-twentieths of the school play soccer. How many learners are there in the school?

16. Adele wrote two tests. She obtained \(\frac{14}{25}\) for the first test and \(\frac{15}{28}\) for the second test.
   a) Convert her test marks to a percentage.
   b) In which test did she perform better?

17. John earns R150 per day. His boss increases the wage by 15%. How much will he earn now?

18. A shop is having a ‘75% off’ sale on all summer items. Xolani buys a dress marked at R250. How much will she pay for the dress?

**Activity 3.5 Using fractions as coefficients in algebraic expressions and equations**

A coefficient is the numeric value in front of an unknown variable in an algebraic expression, for example:
- the coefficient of \(6x\) is 6, \(x\) is the unknown variable here.
- the coefficient of \(\frac{1}{2}y\) is \(\frac{1}{2}\), \(y\) is the unknown variable here.

Simplify the following expression: \(\frac{1}{2}a + \frac{1}{3}a\)

1. The first rule here is to find a LCD. What is the LCD of the denominators?

2. Now we can rewrite this expression as: \(\frac{3}{6}a + \frac{2}{6}a = \frac{5}{6}a\) (note here that what we originally asked for is what is half of \(a\) plus a third of \(a\) and we get \(\frac{5}{6}\) of \(a\)).
   Which property of rational numbers allows us to write \(\frac{3}{6}a + \frac{2}{6}a\) as \(\left(\frac{3}{6} + \frac{2}{6}\right)a\)?

3. Simplify the following expression: \(1\frac{2}{3}b + \frac{6}{5}b - \frac{1}{10}b\)

4. Solve for \(x\) in the equation \(\frac{2}{3}x = 10\).

**Key ideas**

- The first rule here is to convert any mixed number to an improper fraction:
  - \(\frac{5}{2}b + \frac{6}{5}b - \frac{1}{10}b\)
- The LCD here is 30.
Remember to use the multiplicative inverse (reciprocal) of \( \frac{a}{b} \) in the equation \( \frac{a}{b}x = c \) to solve this equation, so \( b \times \frac{a}{b}x = \frac{b}{a} \times c \). This means \( x = \frac{bc}{a} \).

**Worked example**

Work out the following:
\[ \frac{1}{2}(a + b) + \frac{2}{3}(a - b) \]

**SOLUTION**

Multiply the fractions with the expressions in the brackets:
\[ \frac{1}{2}a + \frac{1}{2}b + \frac{2}{3}a - \frac{2}{3}b \]

Group the fractions with \( a \) together and the ones with \( b \) together:
\[ \frac{1}{2}a + \frac{2}{3}a + \frac{1}{2}b - \frac{2}{3}b \]

The LCM here is 6. Rewrite this expression:
\[ \frac{3}{6}a + \frac{4}{6}a + \frac{3}{6}b - \frac{4}{6}b = \frac{7}{6}a - \frac{1}{6}b \]

(Note that we had to work with \( a \) separate from \( b \) as those two variables are different.)

**Worked example**

Consider the following equation and solve for \( a \):
\[ \frac{4}{7}a = 1 \]

**SOLUTION**

\( \frac{4}{7} \) multiplied by something (in this case \( a \)) should give us 1. Hence our solution for \( a \) would be \( \frac{7}{4} \).

Alternatively we could use the rule of equations and multiply both sides by \( \frac{7}{4} \):
\[ \left( \frac{7}{4} \right) \left( \frac{4}{7} \right) a = \left( \frac{7}{4} \right) 1 \]

\[ a = \frac{7}{4} \]

and we get the same answer via calculation.

**Worked example**

Consider the following equation and solve for \( b \):
\[ 2b = 1 \]

**SOLUTION**

Looking at the equation we can see that if \( b \) equals \( \frac{1}{2} \), then this equation holds true. In this example we can either divide both sides of the equation by 2 or multiply both sides of the equation by the reciprocal of 2 which is \( \frac{1}{2} \) (remember 2 can be written as \( \frac{2}{1} \) therefore its reciprocal is \( \frac{1}{2} \)).
\[ \left( \frac{2}{1} \right) b = 1 \left( \frac{1}{2} \right) \]

\[ b = \frac{1}{2} \]
Worked example
Solve for $c$:
\[ 2c + \frac{1}{2}c = 3 \]

**SOLUTION**
The LCM is 2 for the coefficients of $c$ and we get:
\[ \frac{4}{2}c + \frac{1}{2}c = \frac{5}{2}c \]
\[ \frac{5}{2}c = 3 \]

Multiply both sides of the equation with the reciprocal of the coefficient of $c$, in this case it is $\frac{2}{5}$.
\[ \left(\frac{2}{5}\right)\left(\frac{5}{2}c\right) = 3 \left(\frac{2}{5}\right) \]
\[ c = \frac{6}{5} \]

**Exercise 3.4** Fractions as coefficients in algebraic expressions

1. Work out the expression $\frac{1}{2}a(4a - 8) - a^2 + a$.
2. Work out $\frac{3}{4}(x + y) - \frac{2}{3}(x - y)$.
3. Simplify the expression $\frac{1}{8}p + \frac{1}{4}p(2\frac{1}{2} - 3\frac{3}{8})$.
4. Simplify $\left(\frac{m}{2} + \frac{n}{6}\right)\left(\frac{2}{q}\right)$.
5. Show that $\frac{4}{5}(a + b)a - \frac{4}{5}(a + b)b$ is equal to $\frac{4a^2 - 4b^2}{5}$.
6. Solve the equation $3\frac{2}{5}x = 17$.
7. Find the value of $n$ that will make the statement $\frac{1}{4}n + 4n = 85$ true.
8. Solve for $y$ in the equation $\frac{2y - 1}{14} = \frac{15}{7}$.
9. If $x = \frac{a}{3}$, find the value of $\frac{a}{x}$.
10. Find the value of $y$ if $y = \sqrt{\frac{64a^2}{36} + \frac{a^2}{3}}$.
11. The sum of a number and double its value is 225. What is the number?
Summary

- A proper fraction is a fraction whose numerator is smaller than the denominator.
- An improper fraction is a fraction whose numerator is greater than the denominator.
- A mixed number or fraction has a whole number and a fractional part.
- To convert an improper fraction to a mixed number, divide the numerator by the denominator, write the quotient as the whole number and the remainder as the numerator over the divisor.
- To convert a mixed number to an improper fraction multiply the whole number by the denominator and add its numerator over the denominator.
- When adding or subtracting fractions, we have to ensure that the denominator of all the fractions involved in the calculation are the same.
- To add or subtract mixed numbers, we first convert them to improper fractions.
- When multiplying or dividing fractions, it is not necessary to have the same denominator with all the fractions involved in the problem.
- During multiplication of fractions, we multiply the numerator with the numerator and the denominator with the denominator.
- When dividing into a fraction, it is the same as multiplying with its reciprocal.
- Percentages are another form of fractions and can easily be converted into the fraction form provided. Whatever the percentage value we are working with in this range, we just divide it by 100 and leave it in a fraction form.
- The same principles apply when solving or simplifying algebraic expressions that have fractions as coefficients as when the coefficients are whole numbers or decimals.
- The use of the reciprocal or multiplicative inverse is an invaluable tool in solving equations.

Check what you know

1. Reduce the following to its simplest form:
   a) \( \frac{3}{9} \)  
   b) \( \frac{16}{72} \)  
   c) \( \frac{21}{7} \)  
   d) \( \frac{13}{169} \)

2. Express the following as an improper fraction:
   a) \( 1\frac{1}{3} \)  
   b) \( -5\frac{7}{8} \)  
   c) \( 4\frac{3}{4} \)  
   d) \( -5\frac{5}{8} \)

3. Write the following as mixed numbers:
   a) \( \frac{23}{8} \)  
   b) \( \frac{11}{3} \)  
   c) \( -\frac{9}{2} \)  
   d) \( -\frac{122}{11} \)
4. Express the following with a denominator of 40:
   a) \( \frac{1}{2} \)  
   b) \( \frac{3}{8} \)  
   c) \( -\frac{1}{4} \)  
   d) \( -\frac{4}{10} \)

5. Solve the following:
   a) \( \frac{4}{5} + \frac{4}{5} \)  
   b) \( \frac{8}{5} - \frac{4}{5} \)  
   c) \( \frac{3}{5} + \frac{2}{7} \)  
   d) \( 1\frac{2}{3} + 2\frac{3}{4} \)  
   e) \( 3\frac{3}{4} - \frac{5}{6} \)  
   f) \( \frac{5}{7} \times \frac{1}{4} \)  
   g) \( 1\frac{1}{3} \times \frac{5}{8} \)  
   h) \( 2\frac{3}{4} \div \frac{2}{3} \)  
   i) \( 1\frac{1}{2} + \frac{4}{5} \times 2 \)  (remember the BODMAS rule here)
   j) \( \frac{8}{3} \div 3 - \frac{81}{9} \)  (again BODMAS)
   k) \( \left( \frac{1}{5} + \frac{2}{5} \right) \times \frac{2}{3} \)  
   l) \( \frac{2}{3} + \frac{2}{5} + \frac{5}{6} \div \frac{1}{3} \)  

6. Simplify the following expressions:
   a) \( \frac{3}{4}b + \frac{5}{4}b \)  
   b) \( \frac{3}{5}a - \frac{4}{5}a \)  
   c) \( \frac{1}{3}a + \frac{1}{2}a - \frac{1}{8}a \)  
   d) \( \frac{2}{3}(c - d) - 2\frac{1}{5}(d - c) + \frac{1}{4}(4a) \)  

7. Solve for \( a \), \( b \) and \( c \):
   a) \( 2b = \frac{1}{5} \)  
   b) \( \frac{3}{4}a = 6 \)  
   c) \( 1\frac{1}{3}b - \frac{3}{4}b = \frac{2}{24} \)  
   d) \( 10 - \frac{3}{5}c = 2 \)  

8. A Grade 9 class has 34 learners and one teacher (a total of 35 people in the room). The teacher writes on the board that 20% of the total number of people in the room are boys.
   a) How many boys are in the classroom?
   b) If six of the learners in the room are between 13 and 14 years old, what percentage of the total will this be?
   c) In this class a learner scored \( \frac{13}{20} \) for the first test and \( \frac{16}{40} \) for the second test. Work out the percentage scored for each test.

9. A workers’ union demands a 10% increase in salary for all its members. The company is willing to give only a 6% increase.
   a) If most of the workers earn R5 000 per month, work out their new salary after a 6% increase.
   b) How much more will the workers earn each month if they get a 10% increase?
   c) If the union and the company compromise on a 7.5% increase, work out the new salary of most of the workers.
10. A shop is offering a 50% discount on accessories and a 30% discount on clothes. **Note:** accessories are hats, belts, jewellery, etc.

   a) What will a shirt that is marked at R240 cost?
   
   b) What will a hat marked at R60 cost?
   
   c) Zizi buys the following items (the original prices are in brackets):
      - dress (R260), belt (R70), jersey (R300) and earrings (R50).
      What is her total bill?

11. A brand new car costs R150 000.

   a) It will lose 10% of its value each year. What will its value be after four years?
   
   b) If after four years the car is sold at a give-away price of R67 500, what percentage is this of the original amount?

12. The motorbike Honda RC31 Bros has a fuel capacity of 12 litres.

   If the rider starts with a full tank of petrol, on a journey of 120 km:
   
   a) How many kilometres will \(\frac{2}{3}\) of the journey be?
   
   b) How many litres will \(\frac{3}{4}\) of the petrol be?
   
   c) If the bike uses \(\frac{3}{4}\) of the tank for \(\frac{2}{3}\) of the journey, how many litres will it need for the whole trip?
   
   d) Work out \(\frac{3}{4} \times 120\) and \(12 \times \frac{2}{3}\). Explain how you did this.

13. Convert the following fractions to decimals and percentages:

   a) \(\frac{21}{50}\)
   
   b) \(2\frac{3}{25}\)
   
   c) \(-\frac{3}{8}\)
In this unit you will:

- do calculations with decimal fractions
- revise calculation techniques such as estimation and rounding off
- operate on numbers that involve the squares, cubes, square roots and cube roots of decimal fractions
- change between equivalent forms: common fraction, decimal fraction and percentages of the same number
- solve problems involving decimal fractions.

Getting started Writing decimals

We can think of a fraction \( \frac{a}{b} \) as ‘\( a \) divided by \( b \)’. If you use a calculator, you can find that \( \frac{1}{2} \), or ‘1 divided by 2’ is 0,5.

1. Use division to convert the following list of common fractions to decimals. Round off your answers to three decimal places where necessary:
   
a) \( \frac{3}{4} \)  
b) \( \frac{3}{2} \)  
c) \( \frac{1}{10} \)  
d) \( \frac{5}{7} \)  
e) \( \frac{7}{5} \)  
f) \( \frac{99}{100} \)  
g) \( \frac{10}{11} \)  
h) \( \frac{49}{1} \)  
i) \( \frac{0}{3} \)

2. Write the decimal expansion of the unit fractions from \( \frac{1}{2} \) to \( \frac{1}{17} \). What do you notice? It is a good idea to do this without a calculator.

Key ideas

- When we convert \( \frac{1}{3} \) to a decimal fraction using a calculator we get \( \frac{1}{3} = 0,3333\ldots \), which can be written as \( \frac{1}{3} = 0,\overline{3} \) where the dot on the 3 refers to the fact that the 3 is recurring, i.e. there are an infinite number of threes.
  
  Now, 0,\overline{33} = \frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \ldots \)

- If we convert \( \frac{1}{27} \) to a decimal fraction, we get 0,037037037037… We notice that the ‘037’ is repeating so we can write 0,\overline{037} to show this. Some texts place a dot on each repeating digit instead, i.e. 0,\overline{037}.

- Some decimals such as 0,03125 which is \( \frac{1}{32} \), break off after a finite number of terms.
• For others, such as the decimal equivalent of \(\frac{1}{13}\), the number of terms are infinite and they repeat themselves periodically. If we find a decimal fraction where there is no periodical repetition and the terms in the expansion are random, then we will not be able to represent this decimal as a common fraction with integers as numerator and denominator (e.g. \(\sqrt{2}\) or \(\pi\)). These are not fractions (rational numbers) but irrational numbers.

### Activity 4.1 Revising calculations with decimal fractions

Complete the following exercises to revise the decimal fractions work from Grade 8.

1. Arrange each row of decimals from smallest to biggest:
   - a) 0,35 0,33 0,303 0,033 0,54
   - b) 0,655 0,63 0,638 0,653 0,61
   - c) 0,4582 0,4588 0,4509 0,4005 0,4058

2. Arrange each row of decimals from biggest to smallest:
   - a) 0,43 0,792 0,163 0,433 0,099
   - b) 0,526 0,525 0,521 0,522 0,529
   - c) 0,3754 0,3746 0,3792 0,3064 0,3709

3. Use the method of finding equivalent fractions (multiply numerator and denominator by the same power of ten) to solve the following:
   - a) \(6 \div 0,3\)
   - b) \(7,5 \div 0,25\)
   - c) \(1,44 \div 0,12\)
   - d) \(6,4 \div 0,16\)
   - e) \(0,24 \div 0,08\)
   - f) \(2,375 \div 0,25\)

4. Do the following division sums and round off your answer to two decimal places in each case:
   - a) \(6,8832 \div 1,2\)
   - b) \(14,177 \div 0,5\)
   - c) \(5,6952 \div 0,9\)

5. Solve:
   - a) \(2 \times (-0,4)\)
   - b) \(0,2 \times (-0,4)\)
   - c) \((-0,02) \times (-0,4)\)
   - d) \((-0,2) \times (-0,04)\)
   - e) \((-0,16) \div 0,02\)
   - f) \((-3,2) \div (-0,8)\)

6. Work out the following:
   - a) \((12)^2\)
   - b) \((1,2)^2\)
   - c) \((0,12)^2\)
   - d) \(\sqrt{225}\)
   - e) \(\sqrt{2,25}\)
   - f) \(\sqrt{0,0225}\)

7. Convert these fractions to decimals. Use the calculator only when you cannot manage using any other method. Round off your answers to three decimal places when you use the calculator.
   - a) \(\frac{15}{25}\)
   - b) \(\frac{12}{20}\)
   - c) \(\frac{5}{13}\)
   - d) \(\frac{4,24}{200}\)
8. Use your knowledge of decimal fractions and algebra to simplify the following expressions:
   a) \(0.3a + 0.7a\)
   b) \(1.2b - 0.4b\)
   c) \(2.4c^2 + 0.9c^2 - 1.3c^2\)
   d) \(0.7(d - 2)\)
   e) \((x - 0.5)(x + 0.3)\)
   f) \(\frac{0.6y^2}{0.2y^2}\)

Key ideas

Rounding off decimals

We usually round off to make the size of the number more manageable. The answer you get from a calculator is often far more accurate than you need. Decimal numbers can be rounded off in just the same way as whole numbers: you decide what decimal place you want to round to, and then look at the next digit on the right. If the next digit is 5 or more, round up. If the next digit on the right is 4 or less, round down.

Estimating

Rounding off numbers is a useful skill when you want to do rough calculations (estimations) in your head. If you round off the numbers you are working with so that they can be easily used for mental arithmetic, you can get a rough answer to the calculation, and sometimes that is all you need in a practical situation. Another good reason for being able to estimate an answer is to be able to check that the answer you get from your calculator makes sense. It is very easy to make a slip entering numbers into a calculator, and you need to get into the habit of estimating an answer as a check.

Worked example

1. Round 3,7852 off to two decimal places.
   SOLUTION
   The third place is 5, so round up. The second place was 8, so it becomes 9. The number becomes 3,79 (rounded off to two decimal places).

2. How long does it take to travel 270 km at 110 km/h?
   SOLUTION
   Distance = speed \times time. Time = \(\frac{270}{110} = 2.4545454\) hours (check this on your calculator!). The last four or five digits of the answer are not useful – they make the number too detailed. You just want to know that it will take about two and a half hours, so you would round the answer off to the first decimal place, namely 2.5 hours.
Exercise 4.1  Operations with decimals

For each of the following sums:

a) Estimate the answer to the nearest whole number.

b) Calculate the number of decimal places there will be in your answer.

c) Use the method that you are most comfortable with to do the sums.

d) Check your answers against your estimates. Use a calculator to check any answers that are very different from your estimates.

e) Round off your answers to two decimal places.

1. \(2,425 \times 3,65\)  
2. \(8,325 \times 2,64\)  
3. \(12,93 \times 0,42\)  
4. \(5,425 \times 10,75\)  
5. \(30,41 \times 4,57\)  
6. \(14,03 \times 1,23\)

Activity 4.2  Calculations with squares, cubes, square roots and cube roots of decimal fractions

The decimal number 0,4 can be written as \((4 \times 0,1)\) and \(\frac{4}{10}\). Let us see how the properties of powers can be used to find \((0,4)^2\).

1. Important properties of powers:
   - \((a \times b)^n = a^n \times b^n\)
   - \(\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\)

   Use these properties to solve:

   a) \((4 \times 0,1)^2\)  
   b) \(\left(\frac{4}{10}\right)^2\)

   The decimal number 0,04 can be written as \((4 \times 0,01)\) and \(\frac{4}{100}\). Let us see how the properties of powers can be used to find \(\sqrt{0,04}\).

2. Important properties of roots:
   - \(\sqrt[n]{a \times b} = \sqrt[n]{a} \times \sqrt[n]{b}\)
   - \(\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}\)

   Use these properties to solve:

   a) \(\sqrt{4 \times 0,01}\)  
   b) \(\sqrt[2]{\frac{4}{100}}\)

   These properties help us to find the appropriate powers and roots of decimal fractions.
Key ideas

- When a decimal number is being squared, the number of decimal places in the answer doubles. Example: \( (0,03)^2 = 0,03 \times 0,03 = 0,0009 \)
- When a decimal number is being cubed, the number of decimal places in the answer triples. Example: \( (0,03)^3 = 0,03 \times 0,03 \times 0,03 = 0,000027 \)
- In general: \( (a \times b)^n = a^n \times b^n \)

\[ (\frac{a}{b})^n = \frac{a^n}{b^n} \]

- When a decimal number that is a perfect square is square rooted, the number of decimal places in the answer is halved. Example: \( \sqrt{0,0064} = \sqrt{0,08 \times 0,08} = 0,08 \)
- When a decimal number that is a perfect cube is cube rooted, the number of decimal places in the answer is divided by three. Example: \( \sqrt[3]{0,000027} = \sqrt[3]{0,03 \times 0,03 \times 0,03} = 0,03 \)
- In general: \( n \sqrt{\frac{a}{b}} = \frac{n \sqrt{a}}{\sqrt{b}} \)

Worked example

1. \( (0,3)^3 = \left(\frac{3}{10}\right)^3 = \frac{3^3}{10^3} = \frac{27}{1000} = 0,027 \)
2. \( \sqrt[3]{0,064} = \frac{\sqrt[3]{64}}{\sqrt[3]{1000}} = \frac{4}{10} = 0,4 \)

Exercise 4.2  

Squares, cubes and roots of decimals

Calculate the following (without using a calculator):

1. a) \( (0,5)^3 \)
   b) \( (0,03)^3 \)
   c) \( (0,2)^3 + (0,002)^2 \)
   d) \( (1,2)^2 - (0,005)^3 \)
2. a) \( \sqrt[3]{0,027} \)
   b) \( \sqrt{1,44} \)
   c) \( \sqrt[3]{0,064} \)
   d) \( \sqrt[3]{0,03} \times \sqrt{0,81} \)
Activity 4.3 Converting decimal numbers to common fractions

1. A class are asked to write the decimal numbers below as fractions:
   a) 0,25
   b) 0,333...

2. Lebohang writes down her strategy for writing the decimal 0,25 as a fraction:
   ‘Place the decimal digits over the appropriate power of ten and then simplify the fraction’: $0,25 = \frac{25}{100} = \frac{1}{4}$
   a) What do you think Lebohang means by the ‘appropriate’ power of ten?
   b) Why has Lebohang chosen to write 0,25 as $\frac{25}{100}$?

3. Next Lebohang applies her strategy to 0,333...
   a) Explain why Lebohang’s first strategy will not work.
   Lebohang uses equations to help her. She writes:
   Let the equivalent common fraction be $x$. Then $x = 0,333...$
   We can also write $10x = 3,333...$
   Now subtract $x$ from $10x$ and solve for $x$:
   $10x = 3,333...$
   $\quad -x = 0,333...$
   $\quad 9x = 3$
   $\quad x = \frac{3}{9} = \frac{1}{3}$
   b) Explain how this strategy works.

Key ideas

- If the decimal number has one recurring decimal, then we multiply the decimal by 10 and then subtract the original decimal. This strategy has the effect of removing the recurring part of the decimal and allowing us to solve for $x$.
- If the decimal number has more than one recurring decimal, multiply appropriately by the correct power of ten and then subtract and solve for $x$.
- If the decimal fraction is not recurring in some periodical manner, there is no common fraction to replace it and we call it irrational. Sometimes, in the case of irrational numbers, estimates are used, e.g. $\pi \approx \frac{22}{7}$.
Worked example

Find the equivalent common fraction for the decimal 0,121212 ...

**SOLUTION**

\[ 100x = 12,121212... \]
\[ - \ x = 0,121212... \]

\[ 99x = 12 \]
\[ x = \frac{12}{99} = \frac{4}{33} \]

**Exercise 4.3**  Equivalent forms – converting decimals to fractions

1. Convert the following decimal numbers to common fractions:
   - a) 0,75
   - b) 3,2
   - c) 0,005
   - d) 1,01
   - e) 3,25
   - f) 1,375

2. Convert the following recurring decimals to an equivalent common fraction in the simplest form:
   - a) 0,6
   - b) 0,0\overline{9}
   - c) 0,0\overline{37}
   - d) 0,5
   - e) 0,1\overline{3}
   - f) 0,0\overline{3}

**Activity 4.4**  Common fraction, decimal fraction and percentage

Remember that one whole is equal to 100%. It then follows that when converting from decimal fractions to percentages, we simply multiply by 100. When we use the percentage button [%] on a calculator, the calculator does the ‘multiply by 100’ for us. When we convert from percentage to a decimal, we divide by 100.

Complete the conversion table. Simplify the common fraction where you can:

<table>
<thead>
<tr>
<th>Common fraction</th>
<th>Decimal fraction</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\frac{1}{25})</td>
<td>0,04</td>
<td>4%</td>
</tr>
<tr>
<td>2.</td>
<td>0,12</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td>18%</td>
</tr>
<tr>
<td>4. (\frac{7}{20})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>0,8</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td></td>
<td>54%</td>
</tr>
</tbody>
</table>
Summary

Calculations with decimal fractions

- A quick way to multiply decimals is to ignore the decimal comma, do the calculation, and then place the comma correctly afterwards. The number of decimal places in the answer will be the sum of the number of decimal places of the numbers being multiplied.
- When we divide by a decimal, we multiply the fraction by \( \frac{10}{10} \), \( \frac{100}{100} \), \( \frac{1000}{1000} \), etc. until the denominator becomes a whole number. Then we can do the sum using long or short division. For example, \( 1.73 \div 0.234 \) becomes

\[
\begin{align*}
1.73 & = 17.3 \\
0.234 & = 2.34 \\
\frac{173}{234} & = \frac{1730}{234} \\
\end{align*}
\]

- When a decimal number is being squared, the number of decimal places in the answer doubles.
- When a decimal number is being cubed, the number of decimal places in the answer triples.
- In general: \( (a \times b)^n = a^n \times b^n \)
- \( \left( \frac{a}{b} \right)^n = \frac{a^n}{b^n} \)

- When a decimal number that is a perfect square is square rooted, the number of decimal places in the answer is halved.
- When a decimal number that is a perfect cube is cube rooted, the number of decimal places in the answer is divided by three.
- In general: \( \sqrt[n]{a \times b} = \sqrt[n]{a} \times \sqrt[n]{b} \)
- \( \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \)

Calculation techniques

- We usually round off to make the size of the number more manageable. Decimal numbers can be rounded off in just the same way as whole numbers.
- Rounding off numbers is a useful skill when you want to estimate answers. You round off the numbers you are working with so that they can be easily used for mental arithmetic. Another good reason for being able to estimate an answer is to be able to check that the answer you get from your calculator makes sense.
Equivalent forms

- It is not always easy to convert a fraction to a decimal. When the denominator cannot be written in tenths, hundredths, thousandths, etc., it makes sense to use your calculator.
- When we convert $\frac{1}{3}$ to a decimal fraction using a calculator we get $\frac{1}{3} = 0,3333\ldots$ which can be written as $\frac{1}{3} = 0,\overline{3}$, where the dot above the 3 refers to the fact that the 3 is recurring.
- If we convert $\frac{1}{27}$ to a decimal fraction we get $0,037037037037\ldots$. Notice that the ‘037’ is repeating. Write $0,0\overline{37}$ or $0,0\cdot3\cdot7\cdot$ to show the recurring digits.
- Converting a decimal with recurring digits to a common fraction:
  - If the decimal number has one recurring decimal, then we multiply the decimal by 10 and subtract the original decimal. If the decimal number has more than one recurring decimal, multiply appropriately by the correct power of ten and then subtract and solve for $x$.
  - If the decimal fraction is not recurring in some periodical manner, there is no common fraction to replace it and we call it an irrational number. Sometimes, in the case of irrational numbers, estimates are used, e.g. $\pi \approx \frac{22}{7}$.
  - Converting decimal fractions to percentages and vice versa: When converting from decimals to a percentage, we simply multiply by 100. When we convert from percentage to a decimal we divide by 100.

Check what you know

1. Will the answer to $0,06 \times 0,6$ be in hundredths, thousandths or tens of thousandths? Explain your answer.

2. You are given that $1 236 \times 5 781$, is $7 145 316$.
   a) If you multiply $1,236 \times 0,5781$, will your answer be:
      i) 714,5316  
      ii) 0,7145316 
      iii) 7,145316  
      iv) 0,007145316?
   b) Give a reason for your answer.

3. a) Estimate the answer to the nearest whole number for each of the following and then do the sum:
   i) $13,015 + 58,792 \times 12,3$  
   ii) $(0,36 + 2,259) \div 0,03$  
   iii) $(6,325 - 2,0739) \times 3,8$  
   iv) $45,583 - 3,18 \div 1,06$
b) Check your answers against your estimates. Use a calculator to check any answers that are very different from your estimates.

4. Calculate:
   a) \((1,4)^2 \times \sqrt{0,25} \)
   b) \((3,03)^2 + \frac{3\sqrt{0,216}}{2} \)
   c) \(\frac{3\sqrt{0,000216}}{(5,42)^2} \)
   d) \((2,03)^3 - \sqrt{0,0121} \)
   e) \((6,4)^3 \div \frac{3\sqrt{0,064}}{2} \)

5. Write down the fraction equivalents of the following decimal numbers:
   a) 0,71
   b) 0,008
   c) 0,013
   d) 9,3
   e) 0,045

6. Find the decimal and percentage equivalents to the fractions:
   a) \(\frac{86}{50} \)
   b) \(\frac{93}{50} \)
   c) \(\frac{1}{5} \)
   d) \(\frac{435}{1500} \)
   e) \(\frac{432}{300} \)

7. Simplify the following algebraic expressions:
   a) \(4,1x^2 - 0,3x + 0,9x^2 + 2,3x \)
   b) \(0,2x(1,3x - 0,4) \)

8. Jacob drives for 6,25 hours at an average speed of 118,25 km/h. How far does he travel? Round off your answer to three decimal places.

9. Wooden flooring planks are sold for R278,55 per square metre. How much will it cost to floor an area of 27,65 m²? Round off your answer to two decimal places.
In this unit you will:

- revise:
  - how to represent integers in exponential form
  - scientific notation
  - general laws of exponents covered in Grade 8
- work with negative exponents
- perform calculations involving all four operations with numbers in exponential form
- solve real-life problems involving numbers in exponential form and those written in scientific notation.

Getting started  Exponents

1. Write the following in expanded form:
   a) \(3^4\)  
   b) \(a^7\)  
   c) \((-2a^4)^2\)  
   d) \((a^3y^4)^3\)  
   e) \(\left(\frac{2}{3}\right)^4\)

2. Write the following in exponential form:
   a) \(7 \times 7 \times 7 \times 7\)
   b) \(0.3 \times 0.3 \times 0.3 \times 0.3 \times 0.3 \times 0.3\)
   c) \(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\)
   d) \(a \times a \times a \times a \times a \times a\)
   e) \((x \times x \times x) \times (x \times x \times x) \times (x \times x \times x) \times (x \times x \times x)\)

3. Simplify fully:
   a) \(2^3 \times 2^3\)  
   b) \(3^7 \div 3^4\)  
   c) \((10^3)^2\)  
   d) \((-4a^3b^2)^0\)  
   e) \(x^5 \times x^7 \times x^3\)  
   f) \(\frac{x^8}{x^6}\)  
   g) \((3a^3b^4)^3\)  
   h) \(-(2ab^2)^3 \times (-a^2b^2)^4\)  
   i) \(\frac{5a^2(a^2 - a^7)}{10a^4}\)  
   j) \(\frac{-2(3ab^2)^2 + 6a^2b^4}{3a^2b^3}\)

4. Showing all calculations, determine whether the following are true or false:
   a) \(3^2 \times 3^2 = 3^4 \times 3^0\)  
   b) \(3^2 \times 3 = 9^3\)
5. a) Write 5 942 million as an ordinary number:
b) Write 5 942 million in scientific notation.
c) Rewrite the following as ordinary numbers:
   i) \(2,53 \times 10^2\)
   ii) \(5 \times 10^5\)
   iii) \(8,1734 \times 10^{12}\)

d) \(5^6 \div 5^2 = 125\)

e) \((-2a^4b^5)^2 = 4a^8b^{10}\)

f) \((-3a^4b^3)^5 = -15a^{20}b^{15}\)

g) \(\frac{12a^3 + 4a^2}{4a^2} = 3a + 1\)

Key ideas

- Writing a number in exponential form is writing it in a more compact or shorter form: \(8^5 = 8 \times 8 \times 8 \times 8 \times 8\)
- A short way of writing repeated multiplication is to use the exponential form: \(9 \times 9 \times 9 \times 9 \times 9 \times 9 = 9^6\)
- The 8 tells us how many times the 9 must be multiplied by itself.
- When you multiply numbers with the same base, you can add the exponents, for example: \(4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = 4^{2+3} = 4^5\)
- When you divide numbers with the same base, you can subtract the exponents.
- When you raise a power to another power, you multiply the powers together.
- Any base raised to the power of 0 is equal to 1.
- Scientific notation is a shorter way of writing down and comparing big numbers with lots of zeroes.
- \(2,4 \times 10^8\) means \(2,4 \times 10 \times 10 \times 10 \times 10 \times 10\); that is, 2,4 multiplied by 10 five times. This gives you 240 000.

Activity 5.1 Comparing and representing numbers in exponential form

We are going to revise how to represent numbers in exponential form and also how to deal with the comparison of numbers in exponential form.

1. Represent the following in exponential form:
   a) \(14 \times 14 \times 14 \times 14 \times 14 \times 14 \times 14 \times 14 \times 14 \times 14 \times 14 \times 14\)
   b) \(a \times a \times a \times a \times a \times a \times a\)
   c) \(0,3 \times 0,3 \times 0,3\)
2. Write the following in expanded form:
   a) $10^5$
   b) $x^8$
   c) $\left(-\frac{1}{3}x\right)^3$

3. Which of the following numbers is the largest:
   a) $4^2$ or $2^7$? (Hint: Write each number in expanded form and then multiply out.)
   b) $(-2)^4$ or $(-3)^3$?

4. Arrange the following in ascending order: $2^7; 2^4; 2; 2^6; 2^3; 2^0$

**Key ideas**

- $4^5$ means $4 \times 4 \times 4 \times 4 \times 4$. Four is multiplied by itself five times.
- $4^5$ does not mean $4 \times 5$.
- We represent $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ as $2^8$ in exponential form.
- Exponential form is a shorter way of writing a number which is repeatedly being multiplied by itself.
- Descending order is when we arrange numbers from the biggest number to the smallest number.
- Ascending order is when we arrange numbers from the smallest number to the largest number.

**Exercise 5.1 Expanded and exponential form**

1. Represent the following in expanded form:
   a) $3^6$
   b) $\left(\frac{1}{10}\right)^4$
   c) $(-8)^8$
   d) $(x^3y^4)^3$
   e) $(-2a^3b^2)^2$
   f) $4 \times 10^3$
   g) $1,4 \times 10^7$

2. Write the following in exponential form:
   a) $4 \times 4 \times 4$
   b) $\frac{2}{7} \times \frac{2}{7} \times \frac{2}{7} \times \frac{2}{7}$
   c) $0,2 \times 0,2 \times 0,2 \times 0,2 \times 0,2 \times 0,2 \times 0,2$
   d) $x \times x \times x \times x \times x \times y \times y \times y \times y \times y \times y \times y 
   e) $3 \times x \times x \times y \times y \times y \times y \times x \times x \times y \times y \times y \times y \times y \times x \times x \times y \times y \times y \times y \times y \times y 
   f) $8 \times 10 \times 10 \times 10 \times 10 \times 10$
   g) $1,4 \times 10 \times 10 \times 10$
3. Simplify fully:
   a) $10^3$  
   b) $2^5$  
   c) $3^4$  
   d) $8^0$  
   e) $12^2$  
   f) $3 \times 10^4$  
   g) $8,12 \times 10^2$

4. Write down the smaller number of the two numbers given:
   a) $2^5$ or $2^2$  
   b) $(\frac{1}{5})^3$ or $(\frac{1}{5})^4$  
   c) $(-3)^8$ or $(-3)^3$  
   d) $(0,5)^2$ or $0,5$  
   e) $3^2$ or $2^3$  
   f) $2^8$ or $3^4$  
   g) $5^2$ or $2^5$  
   h) $8^2$ or $6^2$  
   i) $6^3$ or $4^4$  
   j) $5 \times 10^5$ or $4 \times 10^6$

5. Arrange the following in descending order:
   a) $4^5; 4^8; 4^4; 4^2; 4^9$  
   b) $(\frac{1}{2})^2; (\frac{1}{2})^3; (\frac{1}{2})^4$  
   c) $(-3)^3; (-3)^6; (-3)^0; (-3)^5; (-3)^2$  
   d) $2^3; 3^2; 4; 5^2$  
   e) $3^5; 8^2; 4^4; 5^3; 2^9$

Activity 5.2  
**Problem solving with decimals raised to powers**

1. Mabotse says that out of the following numbers $(0,2)^0$ is the largest, whereas Emma says that $(0,2)^5$ is the largest. With whom do you agree? Explain why you think this is so. $(0,2)^3; (0,2)^5; (0,2)^2; (0,2)^4; (0,2)^6$

2. There is a long-standing challenge that a piece of paper cannot be folded in half more than seven or eight times.
   a) Take a piece of paper and see how many times you can fold it in half.
   b) What could you do to the size of the paper to get it to fold more times than you have managed to do?
   c) Could you fold it in different directions to get it to fold more than seven or eight times?
   d) What happens to the size of the paper when you fold it?
   e) Work out the size of the paper after two folds if the original size of the paper was 1 024 cm$^2$.
   f) Work out the size of the paper after three folds if the original size of the paper was 1 024 cm$^2$.
   g) Work out the size of the paper after four folds if the original size of the paper was 1 024 cm$^2$.
h) Work out the size of the paper after eight folds if the original size of the paper was 1 024 cm².

i) Discuss with a friend whether the following is true:
   If the size of the original piece of paper is \(x\), then after one fold the size will be \(\frac{x}{2}\) and after two folds the size will be \(\frac{x}{4}\).

j) Develop a number sentence to represent the size of a piece of paper when folded eight times. Let the size of the original piece of paper = \(x\).

**Key ideas**

- Using the laws of exponents makes it quicker to simplify expressions with exponents.
- The laws that you learnt in Grade 8 were:
  - \(2^3 \times 2^2 = 2^{3+2} = 2^5\)
    → when we multiply like bases, we add their exponents.
  - \(2^3 \div 2^2 = 2^{3-2} = 2\)
    → when we divide like bases, we subtract their exponents.
  - \((3^4)^3 = 3^{4 \times 3} = 3^{12}\)
    → when we raise a power to another power, we multiply the powers together.
  - \((2^2a^3b)^4 = 2^{2 \times 4}a^{4 \times 3}b^{3 \times 4} = 2^8a^{12}b^{12}\)
    → everything inside the bracket must be raised to the power.
  - \(4^0 = 1\)
    → any base raised to the power 0 is equal to 1.

**Worked example**

Simplify fully:

- **a) \(a^8 \times a^4 \times a^3\)**
  - \(a^8 \times a^4 \times a^3 = a^{8+4+3} = a^{15}\)

- **b) \(\frac{a^{10}}{a^4}\)**
  - \(\frac{a^{10}}{a^4} = a^{10-4} = a^6\)

- **c) \(\frac{a^{10} - a^6}{a^4}\)**
  - \(\frac{a^{10} - a^6}{a^4} = a^6 - a^2\)

- **d) \((a^3b^4)^5\)**
  - \((a^3b^4)^5 = a^{3 \times 5}b^{4 \times 5} = a^{15}b^{20}\)

- **e) \((-2\sqrt{2})^3\)**
  - \((-2\sqrt{2})^3 = -8 \times 2 \times 2 = -32\)

- **f) \(\frac{-2(3)^2 - 4^2(3)^0 + 2}{(-2)^5}\)**
  - \(\frac{-2(3)^2 - 4^2(3)^0 + 2}{(-2)^5} = \frac{-2 \times 9 - 16 \times 1 + 2}{-32} = \frac{-18 - 16 + 2}{-32} = \frac{-32}{-32} = 1\)
Worked example
Simplify: $2^{-3}$

SOLUTION
Before we can simplify the expression $2^{-3}$, we need to understand what a negative exponent means.

Take an expression which needs simplification:

$$\frac{2^2}{2^5} = \frac{2 \times 2}{2 \times 2 \times 2 \times 2} = \frac{1}{2 \times 2 \times 2} = \frac{1}{8}$$

Another way to simplify the expression is to subtract the exponents as the bases are the same:

$$\frac{2^2}{2^5} = 2^{2-5} = 2^{-3}$$

From the above we can say that $\frac{1}{2^3} = 2^{-3}$.

So $2^{-3} = \frac{1}{2^3}$.

Worked example
Simplify fully:

a) $5^{-2}$

b) $a^{-5}$

c) $\frac{1}{3^{-2}}$

d) $2^2 \div 2^4$

e) $4^{-2}a^2b^3$

f) $(2ab^4)^{-2} \times 5$

SOLUTION

a) $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$

b) $a^{-5} = \frac{1}{a^5}$

c) $\frac{1}{3^{-2}} = \frac{3^2}{1} = 9$

d) $2^2 \div 2^4 = 2^{2-4} = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

e) $4^{-2}a^2b^3 = \frac{a^2b^3}{4^2} = \frac{a^2b^3}{16}$

f) $(2ab^4)^{-2} \times 5 = \frac{1}{(2ab^4)^2} \times 5 = \frac{5}{4a^2b^8}$

Key ideas

- We cannot cancel over a $+$ or $-$ sign, only over $\times$ and $\div$ signs.
- Order of operations need to be used when simplifying expressions:
  1 – Exponents
  2 – Brackets
  3 – Multiplication and Division
  4 – Addition and Subtraction.
- To simplify a negative exponent, we change the location of its base.
  - a base with a negative exponent may be changed to a base with a positive exponent as shown here:
    $$a^{-m} = \frac{1}{a^m}$$
    becomes a denominator
if a base with a negative exponent is a denominator we write it with a positive exponent as follows:

\[
\frac{1}{a^{-m}} = a^{m+n}
\]

negative becomes positive
becomes a numerator

Exercise 5.2 Working with exponents

1. Simplify fully:
   a) \(3^2 \times 3^3\)
   b) \(2^8 \times 2^2\)
   c) \(a^3 \times a^4 \times a\)
   d) \(x^3y^4 \times x^4y^4 \times xy\)
   e) \(2x^3y \times x^2y^4 \times 3xy^3\)
   f) \(5^2x^4 \times 5x^3y^4\)
   g) \((-2)^3a^5 \times (-2)^2a^7\)
   h) \(2x^4y^3 \times xy^2 + 3x^2y^5 \times x^3\)
   i) \(1^{10}x^3 \times (-1)^5x^8\)
   j) \(8^5 \times 8^3 \times 8\)

2. Simplify fully:
   a) \(8^{12} \div 8^7\)
   b) \(2a^8 \div 4a^6\)
   c) \(a^7 \div a^4 \div a^2\)
   d) \(a^{10} \div a^6 \div a^2\)
   e) \(27p^3q^2 \div 9p^q^3\)
   f) \(\frac{5a^4(a^2 - 1)}{5a^2}\)
   g) \(\frac{8m^2(m^5 - mn + 1)}{4m^6}\)
   h) \(\frac{7p^2(p^2 + 2p + 1)}{14p}\)

3. Simplify fully:
   a) \((2^3)^3\)
   b) \((3^2)^2\)
   c) \((10^4)^3\)
   d) \((a^7)^2\)
   e) \((a^2b^4)^3\)
   f) \((-2a^4b^5)^4\)
   g) \((x^5y^2)^2 \times (x^4y^3)^3\)
   h) \((4x^2y^3)^4\)
   i) \((-10x^7y^4)^3\)
   j) \((a^7b^8c^9)^{10}\)

4. Simplify fully:
   a) \(2^{-3}\)
   b) \(10^{-5}\)
   c) \(3^{-4}\)
   d) \(5^{-2}\)
   e) \(1^{-10}\)
   f) \(a^{-3}\)
   g) \(6^{-2}\)
   h) \(\frac{1}{3^{-3}}\)
   i) \((ab)^{-2}\)
   j) \(\frac{3^{-2}}{4^{-1}}\)

5. Showing all calculations, determine whether the following statements are true or false.
   a) \(3^2 \times 3^2 = 9^2\)
   b) \((2x)^{-2} = \frac{1}{4x^2}\)
   c) \(2x^{-2} = \frac{1}{2x^2}\)
   d) \(5^7 \div 5^6 = 1\)
   e) \((-4x^3y^4)^2 = 64x^6y^{12}\)
6. Simplify fully:
   a) $4^2 \times 4$
   b) $(3^2)^3$
   c) $10^8 \div 10^3$
   d) $4^{-3}$
   e) $a^{-10}$
   f) $(a^2b^4)^4$
   g) $2(x^3y^3)^{-3}$
   h) $(-2)^3 \times (-3)^2$
   i) $\frac{2(3)^3 - 4(2)^2 - (3)(2)}{2^3}$
   j) $2^3 \div 2^{-3}$
   k) $(4x)^2 \times (2x)^{-4} \times x^0$
   l) $10x^{-2} \times 10^{-1}x^4$
   m) $\frac{4p^2 - p^3}{p^2}$
   n) $\frac{4p^2 - p^3}{p^2}$
   o) $2a^3(a^4 - 3a^3 + 2a^2 - 1)$
   p) $x^2(-2x^3 - 3x + 1) - x^2(x^3 - 3x^2 + 2x)$
   q) $(15x^3)^{-1} \times 30(x^4)^3$
   r) $\frac{(2^4 - 4^3 + 8)^2}{4^x}$
   s) $\frac{a^5}{a^2} \div \frac{a^3}{a^{-2}} \times \frac{(a^3)^4}{(-a^2)^3}$
   t) $x^2y^{-4} \times (x^4y^2)^{-2} \div \frac{1}{x^2y^{-3}}$

---

## Activity 5.3 Using patterns to investigate negative exponents

The following pattern is given:

<table>
<thead>
<tr>
<th>Base 2</th>
<th>Base 3</th>
<th>Base 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^5$</td>
<td>$3^5$</td>
<td>$5^5$</td>
</tr>
<tr>
<td>$2^4$</td>
<td>$3^4$</td>
<td>$5^4$</td>
</tr>
<tr>
<td>$2^3$</td>
<td>$3^3$</td>
<td>$5^3$</td>
</tr>
<tr>
<td>$2^2$</td>
<td>$3^2$</td>
<td>$5^2$</td>
</tr>
<tr>
<td>$2^1$</td>
<td>$3^1$</td>
<td>$5^1$</td>
</tr>
<tr>
<td>$2^0$</td>
<td>$3^0$</td>
<td>$5^0$</td>
</tr>
<tr>
<td>$2^{-1}$</td>
<td>$3^{-1}$</td>
<td>$5^{-1}$</td>
</tr>
<tr>
<td>$2^{-2}$</td>
<td>$3^{-2}$</td>
<td>$5^{-2}$</td>
</tr>
<tr>
<td>$2^{-3}$</td>
<td>$3^{-3}$</td>
<td>$5^{-3}$</td>
</tr>
<tr>
<td>$2^{-4}$</td>
<td>$3^{-4}$</td>
<td>$5^{-4}$</td>
</tr>
</tbody>
</table>

1. Complete the pattern by simplifying the expressions.

2. Look at the answers in the column which has the base 2. What do you notice about how they progress?