Clever Keeping
Maths Simple

Grade 8
Learner’s Book

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Introduction

Welcome to this Mathematics Grade 8 Learner’s Book.

Have you ever wondered how Mathematics could be useful to you? Do you think that you could use Mathematics outside the classroom?

Do you know that Mathematics is used in different ways every day? For example:
Mathematics is used in MUSIC.
Mathematics is used in MANY TECHNICAL CAREERS.
Mathematics is used in COOKING YOUR FAVOURITE FOOD.
Mathematics is used in COMPUTERS, CELL PHONES AND GAMES.
Mathematics is used when GOING TO THE SHOP TO BUY YOUR FAVOURITE THINGS.

Just about everything we do uses Mathematics! So we can say that Mathematics is a language that makes use of symbols and notations to describe numerical, geometrical and graphical relationships in order to help us to give meaning to the world we live in.

To help you to develop the essential mathematical skills that you need to deal with mathematical situations competently, this Mathematics Grade 8 Learner’s Book will guide you to:
• develop the correct use of the language of Mathematics
• develop number vocabulary, number concepts and calculations and application skills
• communicate, think, reason logically and apply the mathematical knowledge gained
• investigate, analyse, represent and interpret information
• pose and solve problems
• build an awareness of the important role that Mathematics plays in real-life situations.

This Mathematics Grade 8 Learner’s Book covers five main content areas:
• Numbers, Operations and Relationships
• Patterns, Functions and Algebra
• Space and Shape (Geometry)
• Measurement
• Data Handling

The authors and publisher wish you all the best in your study of Mathematics in Grade 8.
In this topic you will learn to:

- revise multiplication of whole numbers to at least 12 × 12
- revise the ordering and comparing of whole numbers
- revise the properties of whole numbers
- discover the division property of 0
- revise calculations with all four operations on whole numbers
- revise and use the following calculation techniques:
  - Rounding off and compensating
  - Estimation
  - Addition, subtraction and multiplication in columns
  - Long division
  - Calculator
- revise prime factors of numbers to at least 3-digit whole numbers
- find the LCM and HCF of numbers to at least 3-digit whole numbers
- solve problems that involve whole numbers, including ratio and rate
- solve problems that involve percentages and decimal fractions in financial contexts.

What you already know

1. Do the following calculations. Do not use a calculator:
   
   a) \(9 \times 8\)  
   b) \(90 \times 8\)  
   c) \(90 \times 800\)  
   d) \(11 \times 12\)  
   e) \(11 \times 120\)  
   f) \(110 \times 1200\)  
   g) \(13 \times 9\)  
   h) \(13 \times 90\)  
   i) \(130 \times 9000\)

2. Do the following calculations. Check all your answers with your calculator.
   
   a) \(7 \times 24\)  
   b) \(16 \times 12\)  
   c) \(9 \times 28\)  
   d) \(22 \times 9\)  
   e) \(15 \times 15\)  
   f) \(11 \times 220\)

3. Arrange the following numbers in:
   
   a) ascending order (smallest to largest): 45 653; 32 654; 45 632; 23 456
   b) descending order (largest to smallest): 4 300; 6 241; 510; 62 410; 43 000; 200; 50 101

4. Make the number sentences true by filling in >, < or =.
   
   a) Five hundred and eight \(\square\) five hundred and eighty
   
   b) Three thousand four hundred and ten \(\square\) three thousand and ten
   
   c) One hundred thousand one hundred \(\square\) one hundred thousand and eighty
   
   d) Five million eight hundred and sixty thousand \(\square\) five million eight thousand and sixty
   
   e) 5 826 \(\square\) 58 269
   
   f) 22 893 \(\square\) 22 899
   
   g) 4 567 200 \(\square\) 4 567 020
Unit 1 Properties of whole numbers

Introduction
Each one of you has a unique place and role in the world. You belong to groups of people who have special relationships with each other. For example, you are a member of your family, whom you live with. Your family is also a member of the community in your area.

Community
You and your family

In mathematics, we group numbers in a similar way to the groups of people. The first group is the group of natural numbers ($\mathbb{N}$). They start at 1 and continue until infinity. The second group is the whole numbers ($\mathbb{N}_0$). This group consists of all the natural numbers and the number 0.

We can also write these numbers as a list:

$\mathbb{N} = \{1; 2; 3; 4; \ldots\}$

$\mathbb{N}_0 = \{0; 1; 2; 3; 4; \ldots\}$

You should know your multiplication tables by heart to at least $12 \times 12$. When you have to multiply numbers larger than $12 \times 12$, first break the numbers up into smaller numbers. This will make it easier to do the multiplications. Let’s look at an example.

Example
Calculate $15 \times 18$.

Solution

\[
15 \times 18 = 15 \times 6 \times 3 \quad \text{(Remember: } 18 = 6 \times 3) \\
= 90 \times 3 \quad \text{(} 15 \times 6 = 90) \\
= 270 \quad \text{(Much easier to do, isn’t it?)}
\]
Compare whole numbers

Having a proper sense of numbers lets us visualise the size of numbers. This helps us to compare them and to estimate answers. It also gives us a way to understand a problem and to check our calculations.

We can compare whole numbers by using a number line.

Example
Fill in >, < or = to make the number sentence true: 119 □ 114.

Solution
Plot the two numbers on a number line. The number on the right is larger than the number on the left. So: 119 > 114.

We can also compare whole numbers by comparing the individual digits in a number. Remember, each digit has a place and a value. The place could refer to units, tens, thousands, and so on.

Example
Compare and order these numbers from smallest to largest:
45 895; 4 895; 345 958.

Solution

<table>
<thead>
<tr>
<th>HTh</th>
<th>TTh</th>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>5</td>
<td>9</td>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

Smallest

<table>
<thead>
<tr>
<th>Number</th>
<th>Thousands</th>
<th>Hundreds</th>
<th>Tens</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>2nd</td>
<td>4</td>
<td>0</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>3rd</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>0</td>
</tr>
</tbody>
</table>
The thousands are the same: all are 4 thousands.

The 2nd number has 0 hundreds compared to the 3 hundreds of the other two numbers. So the 2nd number is the smallest. Continue to compare the 1st and 3rd numbers.

Here, the 1st number has 0 tens. The 3rd number has 8 tens. So the 1st number is smaller than the 3rd number.

The numbers in descending order are: 4 380; 4 308; 4 083.

Number properties and identity properties

Number properties and identity properties help us to simplify calculations and to develop a sense of numbers.

**Commutative property**

The commutative property (or rule) applies to addition and multiplication. This property lets us change the order of the numbers and still get the same answer. Study the following examples.

**Example**

1. \(917 + 73 = 990\) and \(73 + 917 = 990\) So \(917 + 73 = 73 + 917\).
2. \(3280 \times 3 = 9840\) and \(3 \times 3280 = 9840\) So \(3280 \times 3 = 3 \times 3280\).

In both cases, we swapped the numbers around, but still got the same answer.

**Associative property**

The associative property also applies to addition and multiplication. This property (or rule) allows us to group numbers differently when we add or multiply and still get the same answer. Study the following examples.

**Example**

1. \((350 + 632) + 18 = 982 + 18 = 1000\)
   Or
   \(350 + (632 + 18) = 350 + 650 = 1000\)
   So \((350 + 632) + 18 = 350 + (632 + 18)\).
   Which grouping made the addition easier?

2. \((60 \times 4) \times 3 = 240 \times 3 = 720\)
   Or
   \(60 \times (4 \times 3) = 60 \times 12 = 720\)
   So \((60 \times 4) \times 3 = 60 \times (4 \times 3)\).
**Distributive property**

We say the distributive property applies to multiplication over addition and subtraction. To distribute means to spread out. This property (or rule) allows us to **redistribute** numbers and still get the same answer.

**Example**

1. \[15 \times 32 = 15 \times (30 + 2) \quad (32 = 30 + 2)\]
   \[= (15 \times 30) + (15 \times 2) \quad \text{(Now we apply the 15 to both the 30 and the 2.)}\]
   \[= 450 + 30 \quad = 480\]

2. \[23 \times 18 = 23 \times (20 - 2) \quad (18 = 20 - 2)\]
   \[= (23 \times 20) - (23 \times 2) \quad \text{(Now apply the 23 to the 20 and the 2.)}\]
   \[= 460 - 46 \quad = 414\]

Remember that you may omit the multiplication sign before the bracket, since the bracket means you have to multiply, therefore:

\[2 \times (8 + 2) = 2(8 + 2).\]

**The identity properties**

When we apply an identity property to a number, the number stays the same. There are two identity properties, one for addition and one for multiplication.

<table>
<thead>
<tr>
<th></th>
<th>Additive identity: Adding 0 to any number leaves the number unchanged</th>
<th>Multiplicative identity: Multiplying a number by 1 leaves the number unchanged.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Examples</strong></td>
<td>310 + 0 = 310</td>
<td>739 × 1 = 739</td>
</tr>
<tr>
<td></td>
<td>310 − 0 = 310</td>
<td>739 ÷ 1 = 739</td>
</tr>
</tbody>
</table>

**Inverse operations**

Inverse means opposite. So, an inverse operation reverses the effect of another operation.

- **Addition** and **subtraction** are inverse operations:
  
  If \(460 + 340 = 800\), then \(800 - 340 = 460\) or \(800 - 460 = 340\).  

- **Multiplication** and **division** are inverse operations:
  If \(150 \times 40 = 6000\), then \(6000 \div 40 = 150\) or \(6000 \div 150 = 40\).

We can use inverse operations to check the answers to calculations!

**Example**

What number must be multiplied by 482 to get 5784?

**Solution**

We write: \(\square \times 482 = 5784\)

\[\therefore \square = 5784 \div 482\]

(Use the inverse operation.)

\[= 12\]  
(Use your calculator to find the answer.)

**Learn more about the division property of 0**

What have you already learnt about the number 0?

<table>
<thead>
<tr>
<th>0 is the first whole number: (\mathbb{N}_0 = {0; 1; 2; 3; ...})</th>
<th>We know that 0 is the identity element for addition and subtraction. Examples: (5 \div 0 = 5) and (18 \div 0 = 18)</th>
<th>When we multiply any number by 0, the answer is 0.</th>
</tr>
</thead>
</table>

\[0 \div 7?\]
Suppose we have no apples and we want to divide it between 7 children. The answer is 0 apples each.

\[7 \div 0?\]
This means you have 7 apples and want to divide it between 0 children. This does not make sense! So we say that \(7 \div 0\) is undefined.

Can you see that when you want to divide by 0, it does not make sense and is meaningless? Another way of thinking about \(7 \div 0\) is to think about division as repeated subtraction (see Unit 2). The question is how many times can 0 be subtracted from 7 to get 0?

Let’s see:
\[7 - 0 - 0 - 0 - 0 ...\]
We will never be able to end with 0, so \(7 \div 0\) has no meaning.
Exercise 1

1. Use inverse operations to find the unknown number. (You may use your calculator.)
   a) A number is multiplied by 535 and gives an answer of 7 490. Calculate the number.
   b) How many times does Busi need to deposit R240 to have a total of R15 360 in her bank account?
   c) A certain number is divided by 12 000 and gives an answer of 500. What is the number?

2. a) Can you apply the commutative property to subtraction and division? Use examples to support your answer.
   b) Can you apply the associative property to subtraction and division? Use examples to support your answer.

3. Complete the following by using the commutative or associative property of whole numbers:
   I just don’t understand the difference between the commutative and distributive properties! Just relax; it’s not that difficult...
   a) $3\,600 + 231 = \underline{\hspace{1cm}}$ and $231 + 3\,600 = \underline{\hspace{1cm}}$
   b) $43 \times 21 = \underline{\hspace{1cm}}$ and $21 \times 43 = \underline{\hspace{1cm}}$
   c) $(22 \times 3) \times 10 = \underline{\hspace{1cm}}$ and $22 \times (3 \times 10) = \underline{\hspace{1cm}}$
   d) $63 \times (11 \times 23) = \underline{\hspace{1cm}}$ and $(63 \times 11) \times 23 = \underline{\hspace{1cm}}$  
   e) $2\,201 + \underline{\hspace{1cm}} = 2\,647$ and $446 + \underline{\hspace{1cm}} = 2\,647$
   f) $160 + (40 + 26) = \underline{\hspace{1cm}}$ and $(160 + 40) + 26 = \underline{\hspace{1cm}}$
   g) $13 \times 400 = \underline{\hspace{1cm}}$ and $400 \times 13 = \underline{\hspace{1cm}}$

4. Use the distributive property to complete the following:
   a) $5 \times (9 - 2) = (5 \times \underline{\hspace{1cm}}) - (5 \times \underline{\hspace{1cm}}) = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
   b) $50(80 + 5) = (\underline{\hspace{1cm}} \times 80) + (\underline{\hspace{1cm}} \times 5) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
   c) $8 \times 32 = 8 \times (30 + \underline{\hspace{1cm}}) = (8 \times 30) + (8 \times \underline{\hspace{1cm}}) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
   d) $48 \times 9 = (50 - \underline{\hspace{1cm}}) \times 9 = (50 \times 9) - (2 \times \underline{\hspace{1cm}}) = \underline{\hspace{1cm}} - \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
   e) $12(100 + 80) = (\underline{\hspace{1cm}} \times 100) + (\underline{\hspace{1cm}} \times 80) = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

5. Complete the following to make the sentences true:
   a) $256\,813 \times \underline{\hspace{1cm}} = 256\,813$
   b) $0 + 34\,850 = \underline{\hspace{1cm}}$
   c) $\underline{\hspace{1cm}} + 10\,380 = 10\,380$
   d) $1 \times \underline{\hspace{1cm}} = 346\,782$
   e) $0 \div 80 = \underline{\hspace{1cm}}$
   f) $315 \div 0 = \underline{\hspace{1cm}}$
**Unit 2  Calculations with whole numbers**

In a calculation, we do the operations in the following order:

<table>
<thead>
<tr>
<th>Order</th>
<th>Operation</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Do the calculations in brackets</td>
<td>(16 \times (15 + 5) = 16 \times 20 = 320)</td>
</tr>
</tbody>
</table>
| 2     | Multiplication and division from left to right | \(36 \div 3 \times 4 = 12 \times 4 = 48 (\checkmark)\)  
\(36 \div 3 \times 4 = 36 \div 12 = 3 (\times)\) |
| 3     | Addition and subtraction from left to right | \(6 - 5 + 1 = 1 + 1 = 2 (\checkmark)\)  
\(6 - 5 + 1 = 6 - 6 = 0 (\times)\) |

**Rounding off numbers**

We can estimate an answer by rounding off numbers. This helps to simplify mental calculations.

**Rounding off a number to the nearest 5**

<table>
<thead>
<tr>
<th>If the last digit of the number is:</th>
<th>Then the last digit becomes:</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>3, 4, 5, 6 or 7</td>
<td>5</td>
<td>7 343 becomes 7 345</td>
</tr>
<tr>
<td>0, 1 or 2</td>
<td>0</td>
<td>7 342 becomes 7 340</td>
</tr>
<tr>
<td>8 or 9</td>
<td>0 and the tens become 1 more</td>
<td>7 348 becomes 7 350</td>
</tr>
</tbody>
</table>

**Rounding off a number to the nearest 10, 100, 1 000 and 10 000**

If the digit is 5 or more, the number is rounded up. If the digit is less than 5, the number is rounded down.

<table>
<thead>
<tr>
<th>Round off to nearest:</th>
<th>Use the:</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>Units</td>
<td>5 349 becomes 5 350</td>
</tr>
<tr>
<td>100</td>
<td>Tens</td>
<td>3 483 becomes 3 500</td>
</tr>
<tr>
<td>1 000</td>
<td>Hundreds</td>
<td>341 653 becomes 342 000</td>
</tr>
<tr>
<td>10 000</td>
<td>Thousands</td>
<td>241 653 becomes 240 000</td>
</tr>
</tbody>
</table>

To estimate an answer, we first round the numbers off. Then we work out the answer using the rounded numbers. This rough calculation allows us to guess what the result will be. We can also use this technique to work out two values between which our answer will lie.

**Example**

Between which two values will \(23 \times 76\) lie?
Solution
Round both numbers down: \(20 \times 70 = 1400\)
Round both numbers up: \(30 \times 80 = 2400\)
\(\therefore 23 \times 76\) lies between 1400 and 2400.

Example
Estimate \(72 \times 33\).

Solution
\[
72 \times 33 \approx 70 \times 35 \quad \text{(Round both numbers off to the nearest 5.)}
\]
\[
= (70 \times 30) + (70 \times 5) \quad \text{(Distributive property)}
\]
\[
= 2100 + 350
\]
\[
= 2450
\]

Example
Estimate \(167 \times 84\).

Solution
\[
167 \times 84 \approx 170 \times 80 \quad \text{(Round both numbers off to the nearest 10.)}
\]
\[
= 13600 \quad \text{(Calculate 17 \times 8 and write two zeros at the end.)}
\]

Rounding off and compensating
Rounding off and compensating is another technique that we can use to simplify calculations. This means that we round off numbers to make the calculation easier and then compensate to make up for what will be lost or gained when rounding off numbers.

Examples
Use rounding off and compensation to simplify the following calculations:

1. \[
893 - 56 = (893 + 7) - (56 + 7)
= 900 - 63
= 837
\]
   \[
\quad \text{or} \quad 893 - 56 = (893 + 4) - (56 + 4)
= 897 - 60
= 837
\]

2. \[
58 + 29 = (58 + 2) + (29 - 2)
= 60 + 27
= 87
\]

3. \[
26 \times 15 = (30 \times 15) - (4 \times 15)
= 450 - 60
= 390
\]
   \[
\quad \text{(Rounding off 26 to 30.)}
\]

4. \[
150 \div 3 = (150 \div 3) \div (6 \div 3)
= 50 \div 2
= 52
\]
   \[
\quad \text{(Round off 156 to a number that can easily be divided by 3.)}
\]
**Addition and subtraction**

When we add or subtract numbers, we add or subtract the digits with the same place values. When you add and the answer of the digits is 10 or more, you carry it over the next column on the left. When you subtract, you sometimes need to borrow from the column on the left.

**Example**

Calculate:

1. \(729 + 831\)
2. \(85\,384 - 4\,827\)

**Solution**

1. \(729 + 831\)

\[
\begin{array}{c|c|c|c}
& \text{Th} & \text{H} & \text{T} & \text{U} \\
\hline
+ & 7 & 2 & 9 & \\
+ & 8 & 3 & 1 & \\
\hline
& 1 & 5 & 6 & 0 \\
\end{array}
\]

2. \(85\,384 - 4\,827\)

\[
\begin{array}{c|c|c|c|c}
& \text{TTh} & \text{Th} & \text{H} & \text{T} & \text{U} \\
\hline
\text{–} & 8 & 4 & 8 & 2 & 7 \\
\text{–} & 5 & 3 & 8 & 1 & 4 \\
\hline
& 8 & 0 & 5 & 5 & 7 \\
\end{array}
\]

**Multiplication**

You already know that multiplication is the same as repeated addition.

For example:

\(3 \times 6 = 6 + 6 + 6 = 18\)

Also

\(6 \times 3 = 3 + 3 + 3 + 3 + 3 + 3 = 18\)

We can see this if we draw two tables. The first has three rows of six columns. The second has six rows of three columns. As you can see, both tables contain the same number of squares.
**Multiplication in columns**

Remember to put the digits with the same place values underneath each other.

**Example**
Calculate $273 \times 12$.

**Solution**

<table>
<thead>
<tr>
<th></th>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\times$</td>
<td>2</td>
<td>7</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$+$</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>7</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

Doubling numbers is an interesting way to multiply two numbers.

**Example**
Calculate $18 \times 35$ using the doubling method.

- $1 \times 35 = 35$
- $2 \times 35 = 70$
- $4 \times 35 = 140$
- $8 \times 35 = 280$
- $16 \times 35 = 560$

Now we use the bold digits. We need to add the digits that give us 18. So we add $16 + 2$. To find the answer to $18 \times 35$, we use the answers to $16 \times$ and $2 \times$. So we have:

$18 \times 35 = 560 + 70 = 630$

**Division**

We can write a division sum in different ways:

- $12 \div 6$
- $\frac{12}{6}$
- $6 \parallel 12$

Division is the same as repeated subtraction:

$12 \div 6 = 2$

$12 - 6 - 6 = 0$

We subtract until we reach zero. Here we subtracted 6 from 12 twice. So this means that 6 goes into 12 twice.
Division can also mean sharing. For example, if we divide 12 apples between 6 learners, each will get 2 apples.

Another way to describe this is to say we are dividing 12 apples into groups of 6. Then we will have 2 groups. We call this division grouping.

Next, let's revise how to do long division.

\[
\begin{array}{c|cc}
12 & 4 & 8 \\
\hline
57 & 4 & 8 \\
- 9 & 6 & 96 \\
- 9 & 6 & 0 \\
\hline
0 & & 0 \\
\end{array}
\]

12 cannot go into 5, but it goes 4 times into 57. Write the 4 above the 7 of 57:

\[57 - 48 = 9; \text{ then we bring down the 6.}\]

12 goes 8 times into 96, so we write the 8 above the 6

\[8 \times 12 = 96; \text{ so when we do the subtraction, the answer is 0.}\]

And we’re done.

So \[\frac{576}{12} = 48.\] There is no remainder.
We can also use short division to find an answer:

\[
\begin{array}{c|cc}
\text{Dividend} & 615 \\
\hline
\text{Divisor} & 3 \\
\end{array}
\]

\[
205
\]

- First, 3 goes into 6 twice. So we write a 2 above the 6.
- Then, 3 does not go into 1, so we write a 0 above the 1. We carry the 1 to the next digit.
- Now, 3 goes 5 times into 15.

Therefore, \(\frac{615}{3} = 205\).

**Calculator skills**

Calculators make calculations quick and easy. Just remember, whenever you use a calculator, you still need to show all your workings. It is still possible to make a mistake when using a calculator. For example, you might accidentally key in a wrong value or press an incorrect button.

Another good idea is to estimate the answer before using your calculator. This is another way to check that you have not made a mistake. If you have manually worked out the answer, use your calculator to check the answer.

**Example**

Calculate \(10 \times 20 - 12 \div 2\):

1. without using a calculator
2. by using a calculator.

**Solution**

1. \(200 - 6 = 194\)
2. Key in: ‘10 \times 20 - 12 \div 2 =’
   
   The answer should be 194.
Example

1. Calculate $18 + (16 - 14 \div 2) \times 2$ using your calculator.
2. Check your answer by applying the correct order of operations.

Solution

1. Key in exactly as given and press ‘=’ at the end. The answer is 36.
2. $18 + (16 - 14 \div 2) \times 2 = 18 + (16 - 7) \times 2$
   $= 18 + 9 \times 2$
   $= 18 + 18$
   $= 36$

Exercise 2

1. Calculate the following:
   a) $27 \div 3 + 6$
   b) $27 \div (3 + 6)$
   c) $40 - (5 \times 3 - 8)$
   d) $40 - 5 \times 3 - 8$
   e) $12 - 3 + 7 - 1$
   f) $2 \times 12 \div 4 \times 3$
   g) $254 + 13 - 22 \times 3$
   h) $16 \div 8 \times 43 - 40$
   i) $15 \times 20 - 3 \times 34$
   j) $3 \times (12 + 14) + 2 \times (13 - 7)$
   k) $1792 \div 32 - 50 + 12$
   l) $21 + 21 \times 3 + 45 \times 2$
   m) $(25 + 5) \div (3 + 3)$

2. Copy and complete the table. The first one has been completed for you.

<table>
<thead>
<tr>
<th></th>
<th>Estimated answer</th>
<th>Calculator answer</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>$58 \times 23$</td>
<td>$60 \times 20 = 1200$</td>
<td>134 (under)</td>
</tr>
<tr>
<td>b)</td>
<td>$67 \times 27$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>$42 \times 36$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td>$33 \times 57$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e)</td>
<td>$323 \times 59$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f)</td>
<td>$125 \times 76$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>g)</td>
<td>$341 \times 77$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>h)</td>
<td>$234 \times 178$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>i)</td>
<td>$715 \times 315$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>j)</td>
<td>$616 \div 28$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k)</td>
<td>$265 \div 53$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>l)</td>
<td>$228 \div 19$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>m)</td>
<td>$528 \div 48$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n)</td>
<td>$928 \div 32$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>o)</td>
<td>$588 \div 28$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. For the following questions:
   i. calculate the answer using long division
   ii. check your answer using an inverse operation and calculator.

   a) $747 \div 27$
   b) $385 \div 24$
   c) $5430 \div 15$
   d) $2664 \div 32$
   e) $6754 \div 25$
   f) $2412 \div 18$
4. For the following questions:
   i. use rounding off and compensating to calculate the answer.
   ii. estimate the answers for a) to d) using rounding. Estimate the answers for e) and f) using doubling.
   a) $324 + 68$
   b) $257 + 137$
   c) $94 – 51$
   d) $187 – 78$
   e) $30 \times 36$
   f) $15 \times 44$

5. Check the correctness of this calculation: $152 \times 38 = 5776$

Remember that there are various techniques that you can use to check that your calculation is correct.

Check the answer by:
   a) estimating two numbers between which the answer will lie
   b) using the distributive property
   c) using inverse operations
   d) using rounding off
   e) using a calculator.

Unit 3  Multiples and factors

Multiples of a number are all the numbers into which the number will divide without a remainder. For example, the multiples of 7 are 7; 14; 21; 28; … Can you see that 7 divides into 7, 14, 21 and 28 without a remainder?

Factors are all the numbers that can divide exactly into another number. For example, the factors of 8 are 1; 2; 4 and 8. These numbers all divide into 8 without a remainder.

Divisibility rules

The divisibility rules make it easier to find the factors of whole numbers!
### Divisibility Rules

<table>
<thead>
<tr>
<th>Divisible by</th>
<th>When</th>
<th>Example</th>
<th>Divisible by</th>
<th>When</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>The last digit is even or 0.</td>
<td>86; 520; 764; 2348</td>
<td>7</td>
<td>When double the last digit subtracted from the remaining digits in the number is divisible by 7</td>
<td>357: (35 - 14 = 21) which is divisible by 7</td>
</tr>
<tr>
<td>3</td>
<td>The sum of the digits is divisible by 3</td>
<td>8751 because (8 + 7 + 5 + 1 = 21)</td>
<td>8</td>
<td>The number formed by the last three digits is divisible by 8</td>
<td>8816 ((816 ÷ 8 = 102))</td>
</tr>
<tr>
<td>4</td>
<td>The number formed by the last two digits is divisible by 4</td>
<td>32532 ((32 ÷ 4 = 8))</td>
<td>9</td>
<td>The sum of the digits is divisible by 9</td>
<td>32688 because (3 + 2 + 6 + 8 + 8 = 27)</td>
</tr>
<tr>
<td>5</td>
<td>The last digit is 0 or 5</td>
<td>35; 7660</td>
<td>10</td>
<td>The last digit is 0</td>
<td>30, 600, 78870</td>
</tr>
<tr>
<td>6</td>
<td>Divisible by 2 and 3</td>
<td>76446</td>
<td>11</td>
<td>Add alternate digits (starting at units), then add remaining digits. Subtract. Answer must be 0 or multiple of 11</td>
<td>22704: ((2 + 7 + 4) - (2 + 0) = 13 - 2 = 11)</td>
</tr>
</tbody>
</table>

A **prime number** can only be divided by 1 and itself. We can also say a prime number has only two factors: 1 and itself. Numbers that have more than two factors are called **composite numbers**.

The number 1 has only one factor. So it is neither a prime number nor a composite number.
The **prime factors** are factors of a number that are also prime numbers. You can use a factor tree or prime factorisation to break a whole number into its prime factors.

### Example
What are the prime factors of 24?

#### Solution
*Use a factor tree:*

```
  24
 /  \  
3    8
 / \  / \  
3  2  4
 / \  / \  
3  2  2  2
```

Prime factors: $2^3 \times 3$  

*Use prime factorisation:*

Prime factorisation is also referred to as the ladder method. Here, we repeatedly divide the number by prime numbers. We start with the smallest prime number that we can divide into the number:

```
<table>
<thead>
<tr>
<th>Prime</th>
<th>Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
```

∴ The prime factors for 24 are $2 \times 2 \times 2 \times 3 = 2^3 \times 3$.

We can use prime factors to find the **highest common factor** (**HCF**) of two or more numbers. To find the HCF, we calculate the product of the factors common to all the numbers. The HCF is very useful when simplifying fractions.

### Example
Find the HCF of 12 and 24.

Prime factors of 12 = $2 \times 2 \times 3$  
(Common factors: 2; 2 and 3)

Prime factors of 24 = $2 \times 2 \times 2 \times 3$

∴ The HCF for 12 and 24 is $2 \times 2 \times 3 = 12$. (Multiply the common factors.)

We use prime factors or multiples to find the **lowest common multiple** (**LCM**) of numbers. The LCM is the smallest number that appears in both lists of multiples.
Example

1. Find the LCM of 9 and 12 using multiples.
   Multiples of 9 are 9; 18; 27; 36; 45; 54; ...
   Multiples of 12 are 12; 24; 36; 48; ...
   The LCM of 9 and 12 is the lowest multiple that appears in both sets.
   So the LCM is 36.

2. Find the LCM of 9 and 12 using prime factors.
   Prime factors of 9: 3 × 3
   Prime factors of 12: 2 × 2 × 3
   Count the number of times each prime factor appears in both lists of factors.
   For example:
   Prime factor 2 appears 0 times in 9 but 2 times in 12: so it appears in the LCM 2 times.
   Prime factor 3 appears 2 times in 9 but 1 time in 12: so it appears in the LCM 2 times.
   ∴ The LCM of 9 and 12 is 2 × 2 × 3 × 3 = 36.

Exercise 3

1. Decide whether the following statements are true or false. In each case, justify (give reasons for) your answer:
   a) 6 is a factor of 24
   b) 5 is a prime factor of 45
   c) 26 is a multiple of 4
   d) 30 is divisible by 6
   e) 4 is a factor of 38
   f) 1 is a factor of every whole number

2. Complete the following in your exercise book:
   a) List the factors of 36.
   b) List the factors of 52.
   c) Name two factors of 15 that are also factors of 45.
   d) List the multiples of 5 between 10 and 50.
   e) List the multiples of 3 from 3 to 24.
   f) Write down the first four multiples of 25.
   g) What multiples of 10 are also less than 90?
   h) Name the multiples of 12 that are greater than 36, but less than 72.

3. Use a factor tree to find the prime factors of the following numbers:
   a) 30
   b) 52
   c) 360
   d) 576
4. Find the HCF for each set of numbers. (You may use your calculator.)
   a) 18 and 54  b) 8 and 180  c) 32 and 80  
   d) 63 and 28  e) 8; 12; 28  f) 24; 42 and 60

5. Use multiples to find the LCM of the following numbers:
   a) 6 and 15  b) 24 and 36  
   c) 23 and 69  d) 2 and 9 and 18  
   e) 65 and 260 and 520  f) 24; 36 and 60

---

**Unit 4  Solving problems**

### Important words

- **account**: a credit facility at a shop that allows us to pay for goods over a period of time
- **budget**: a statement of expected income and expenses
- **credit**: a loan
- **discount**: a reduction in the normal price of an item
- **exchange rate**: the cost of a currency in terms of another country’s currency
- **hire purchase**: an agreement by which we pay for an item over over a period of time
- **interest**: the cost of borrowing money
- **loan**: an amount of money we borrow from a bank
- **loss**: when we sell an item for less than we paid for it; decreases our money
- **profit**: when we sell an item for more than we paid for it; increases our money
- **rate**: a comparison of two quantities of different kinds
- **ratio**: a comparison of two quantities of the same kind

### Ratio

We use ratio when we compare two or more quantities of the same kind.

---

**Example**

One job is created in South Africa for every eight tourists who visit the country. Here, we are comparing two quantities: the number of tourists and the number of jobs for people that tourism creates. We can compare these two quantities by expressing them as a ratio.
Many common problems are expressed in the language of ratio and proportion. For example, we can say the following:

- There is 1 new job created by 8 tourists.
- There is 1 job for every 8 tourists.
- The ratio of jobs to tourists is 1 to 8.
- The number of jobs is $\frac{1}{8}$ of the number of tourists.
- There are $\frac{1}{8}$ as many jobs as tourists.

All five sentences give us the same information! In mathematical symbols, we can express a ratio in two ways:

- The ratio of jobs to tourists is $1 : 8$.
- The ratio of jobs to tourists is $\frac{1}{8}$.

**Rate**

We use **rate** to compare two quantities of different kinds.

For example:

- The doctor charges at a rate of R600 per hour (R600/h).
- Drivers are not supposed to drive faster than 120 km/h on the highway.

**Exercise 4**

You may use your calculator, but show all your workings.

For questions 1 to 5, do the following:

- Write down the calculation to solve the problem.
- Estimate the answer.
- Use your calculator to find the answer.

1. A library buys 638 new books. Each shelf in the library can take 32 books. How many shelves are needed?
2. On a citrus farm, a farmer packs oranges in boxes. Each box holds 42 oranges. These boxes are then packed into a larger container that holds 1 874 oranges. How many complete boxes does the larger container hold?
3. The 52 learners in the Grade 8A class each made 28 masks to be sold at the school bazaar. How many masks did these learners make altogether?
4. A primary school teacher bought a box full of counters to use in her Mathematics class. In the box were 32 bags, each containing 95 counters. How many counters were there altogether?

5. Once a month, a school has a fire drill. Learners are lined up in rows of 38. How many rows would there be if there were 798 learners at school on a particular day?

6. Use the HCF to solve these problems:
   a) Asgar has three sticks measuring 9 cm, 12 cm and 18 cm. He wants to cut the three sticks into shorter lengths of all the same size. What is the largest length he can cut them into so that no pieces are left?
   b) A supermarket wants to donate 217 hot dogs and 126 juices to homeless shelters. The hot dogs and juices must be divided equally between the shelters.
      i. What is the greatest number of shelters the supermarket can donate refreshments to?
      ii. How many hot dogs and juices will each shelter receive?

7. We can use ratios to divide total quantities and to increase and decrease quantities. Answer these problems in your exercise book.
   The profit of Jongani Tour Operators is divided among the three directors: A, B and C. The profit is divided into the ratio 4 : 3 : 2. If the profit is R3 600, how much does each director receive?
   (Hint: Director A receives \( \frac{4}{9} \) of the total profit.)
   a) \( \frac{4}{9} \times 3600 = \) \( \frac{4}{9} \times 3600 \) = A receives R_____.
   b) \( \frac{3}{9} \times 3600 = \) \( \frac{3}{9} \times 3600 \) = B receives R_____.
   c) Therefore, C receives R_____.

8. Solve the following ratio problems:
   a) You visit the zoo and watch the elephants eat their food. Divide 560 kg of food between two elephants in the ratio 3 : 4. How much food does each elephant get?
   b) Two restaurant owners buy a bag of onions for R32,00. The bag contains 192 onions. If Owner A contributed R15,00 and Owner B put in R17,00, how many onions should each receive?
   c) Decrease R60 in the ratio 4 : 5.
   d) Increase R340 in the ratio 3 : 2.
   e) A map of South Africa is drawn to a scale of 1 cm to 10 km.
      i. What does this mean?
      ii. The distance on the map is 8,5 cm. What is the distance on the ground?
   f) Tebogo and Lesedi are waiters in a restaurant. Tebogo receives 15c for every 20c that Lesedi receives. How much money will each receive if they have to divide R49,70 between them?
9. Mrs Toka wants to go on a bus tour to Mpumalanga for two weeks. She has saved R8 500. Use your calculator to calculate the following:
   a) How much can she spend per day?
   b) Use the rate you calculated in a) to say how much more she must save if she wants to stay for an extra three days?

10. The speed limit on a road is 80 km/h. If a bus driver drives at a constant speed (without going faster or slower) for 4 hours, how far would the bus have travelled?

11. In Nelspruit, the bus stopped for everyone to watch an afternoon of cricket. The opening batsman scored 180 runs in 2 hours. His partner scored 100 runs in 80 minutes.
   a) Who scored at a faster rate?
   b) What was the difference between their scoring rates?

12. The bus driver is paid R360 for driving for 8 hours. At what rate is he paid?

13. The cost of a sandwich is R4.50. How much would six sandwiches cost?

Finance

Finance is about how to manage our money. Finance includes topics such as profit, loss, discount, VAT, budgets, accounts, loans, simple interest, hire purchase and exchange rates.

• We make a profit when we sell something for more than we paid for it. So a profit increases the amount of money we have.
• We make a loss when we sell something for less than we paid for it. So a loss decreases the amount of money we have.
• A shop offers a discount by selling some products at a lower price than usual. We most often express a discount as a percentage. For example, at the beginning of summer, a shop might offer a 50% discount on all winter clothes.
• If a shop lets people open an account, then customers can buy items on credit. This means that a customer can buy an item now, but pay it off over a period of time. For example, many people have accounts at clothing shops.
• A budget is a plan of your expected income and expenses. So a budget states what you expect to happen in the future. That way, you can plan holidays or other purchases. A budget also helps to prevent unexpected, nasty surprises. For example, you might need to pay for a TV licence in a few months. But you may have forgotten about it, because it is not one of your regular expenses. A budget will help to make sure the money is available when you need it.
Exercise 5
You may use your calculator in this exercise.
1. Solve the following problems based on profit, loss and discount:
   a) Ronald buys a number of articles for R675. To advertise and rent a store costs a further R150. If he sells his articles for R1 250, what is his profit or loss?
   b) The cost price of an article is R315. It is then sold for R250. Calculate:
      i. the loss
      ii. the percentage loss.
   c) Kgomotso buys a car for R84 000 and sells it for R79 000. What is her percentage loss?
   d) Ismael buys a house for R495 000 and sells it for R620 000. What is his percentage profit?
   e) There is a sale at a local shop and everything is marked down by 25%. A pair of jeans is marked R200,00. What will the jeans cost after the discount?

2. Use the following information to design your own invoice for a client.
The labour for a paint job is R850. You bought the following articles to complete the job:
3 paint brushes @ R37,99 each
4 bottles of thinners @ R23,95 each
1 plastic cover @ R69,45 a cover
3 boxes polyfilla @ R36,65 a box
2 × cloths @ R19,99 each

3. a) Copy and complete the following income and expenses statement from January to May.

<table>
<thead>
<tr>
<th></th>
<th>January</th>
<th>February</th>
<th>March</th>
<th>April</th>
<th>May</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income</td>
<td>R16 843</td>
<td>R17 543</td>
<td></td>
<td></td>
<td>R18 449</td>
</tr>
<tr>
<td>Expenses</td>
<td>R15 674</td>
<td>R13 784</td>
<td>R14 233</td>
<td>R17 455</td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td></td>
<td></td>
<td>R530</td>
<td></td>
<td>R397</td>
</tr>
<tr>
<td>Loss</td>
<td></td>
<td></td>
<td>R870</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Was there a profit or loss over the five months?
Learn more about Finance

- **Value added tax** (VAT) is tax the buyer pays on top of the normal price of a product. It is another way the government can collect money. VAT is included in the price of most products and services that we use. Some basic foods, such as bread, milk and mielie meal, are exempt from VAT to help those who earn less to be able to afford food.

- **Interest** is the cost of borrowing money from someone else. If we borrow money, then we pay interest. If we lend money to someone else (including the bank), then they pay us interest.

- A **loan** is a type of debt. The borrower initially receives an amount of money, for example, from a bank. Then, that money needs to be repaid, usually with interest.

- If we do not have enough cash to buy something, then we can sometimes buy it using a **hire purchase** agreement. We receive the item now, but pay for it over a few months. Each monthly payment is called an instalment. Since we are paying for an item over time, it only belongs to us once all the payments are complete. The payments usually include interest.

- An **exchange rate** is essentially the cost of buying cash in another currency. For example, if $1,00 = R8,00, then we must pay R8 for every US dollar we buy. If a person is visiting South Africa from the United States, then R1,00 will cost them $0,13 (rounded off).

### Example: VAT
Saloshna wants to buy a product that costs R350 before VAT. VAT is charged at 14%. Calculate the total cost after VAT is added.

**Solution**

\[
\text{VAT} = \frac{14}{100} \times \frac{350}{1} = R49
\]

The total cost = R350 + R49 = R399.

### Example: Hire purchase
A second-hand car costs R45 000 cash. However, it can be bought on hire purchase. In this case, the buyer must first pay a 10% deposit, which is R4 500. This leaves R40 500 outstanding (still to be paid). The bank lends money at a fixed rate of 10% of the loan. So the buyer would have to borrow the R40 500 at 10% interest.

1. What is the total cost of the car?
2. How much is the monthly instalment if the car is paid off in one year?

**Solution**

1. Interest = \(10 \times \frac{R40 500}{1} = R4 050\)

   Total cost of the car = Deposit + Money borrowed + Interest
   = R4 500 + R40 500 + R4 050
   = R49 050

2. Monthly instalment = (Money borrowed + Interest) \(\div\) 12 months
   = \(\frac{40 500 + 4 050}{12}\)
   = R3 712,50
**Example: Exchange rates**

Each country has its own money system. For example, we use the rand in South Africa, the UK uses the pound (£), Europe uses the euro (€), Japan uses the yen (¥) and the US uses the dollar ($).

Exchange rates help us to convert amounts of money between the different systems. For example, suppose 1 British pound (£) = R13,08. Thiru wants to visit some relatives in London and has saved R15 000 for all his expenses. He went to the bank to exchange his South African rands for British pounds. Use your calculator to find how many British pounds he receives.

**Solution**

\[
R15\,000 \div 13,08 = £1\,146,79
\]

---

**Exercise 6**

You may use your calculator in this exercise.

1. Suppose VAT is 13%. Solve the following problems:
   a) Before adding VAT, an item costs R600. What is the total cost, including VAT?
   b) The cost of using a service is R125, which does not include VAT. What is the cost of the service after including VAT?
   c) Mandy plans to buy a kettle for R150, excluding VAT. What is the total cost of the kettle after VAT is added?

2. Calculate the interest on the following amounts if the interest is set at 5%:
   a) R2 421,00 (for 1 year)
   b) R5 000,00 (for 2 years)
   c) R56 540,21 (for 3 years)

3. A motorcycle costs R25 250 cash. You do not have this amount so you decide to borrow the full amount from your parents at 10% interest per year.
   a) How much would you pay back if you paid the loan back in one year?
   b) How much would you pay back if you paid the loan back in two years?
   c) How much can you save in 24 months if you paid cash for the motorcycle?
4. *Mandy likes new clothes and selects a few dresses, shoes and underwear to buy. The total bill comes to R3 585,26. Which option would be the better deal if she had to pay it off over 12 months?

**Option 1:** The first six months are interest free. After that, she must pay 20% interest on the outstanding amount.

**Option 2:** Pay 9.8% interest for all 12 months on the total amount.

Questions 5–7 are based on the following international money exchange table.

<table>
<thead>
<tr>
<th>Key</th>
<th>USD</th>
<th>ZAR</th>
<th>GBP</th>
<th>EUR</th>
<th>AUD</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States dollar</td>
<td>1 USD</td>
<td>1</td>
<td>8,41249</td>
<td>1,54843</td>
<td>1,2485</td>
</tr>
<tr>
<td>South African rand</td>
<td>1 ZAR</td>
<td>0,118871</td>
<td>1</td>
<td>0,0767686</td>
<td>0,0952109</td>
</tr>
<tr>
<td>British pound</td>
<td>1 GBP</td>
<td>0,645815</td>
<td>13,0262</td>
<td>1</td>
<td>0,8063</td>
</tr>
<tr>
<td>Euro</td>
<td>1 EUR</td>
<td>0,800961</td>
<td>10,503</td>
<td>1,24023</td>
<td>1</td>
</tr>
<tr>
<td>Australian dollar</td>
<td>1 AUD</td>
<td>1,0141</td>
<td>8,13619</td>
<td>1,57026</td>
<td>1,2661</td>
</tr>
</tbody>
</table>

5. How much will you pay in South African rand for the following currency?
   a) 1 USD  
   b) 2 AUD  
   c) 3 GBP  
   d) 1 ZAR  
   e) 12 EUR

6. How much will you pay in US dollars for the following currency?
   a) 5 South African rands  
   b) 1 euro  
   c) 10 Australian dollars

7. Which currency is the:
   a) most expensive for a South African to buy  
   b) cheapest for a South African to buy?
In this topic you will learn to:

- count forwards and backwards in integers
- order and compare integers
- revise the addition and subtraction of integers
- multiply and divide with integers
- do calculations with all four operations
- recognise and use the commutative, associative and distributive properties of numbers
- recognise and use the additive and multiplicative inverses for integers
- solve problems in contexts involving multiple operations with integers.

What you already know

1. Use the number lines to count forwards and backwards as asked. Then, write down the sequence of numbers as you counted.
   a) Count forwards in intervals of 4 from −17 to −1.
   b) Count backwards in intervals of 15 from 10 to −80.
   c) Count forwards in intervals of 12 from −72 to 12.

2. Complete the following chain diagram. First, count backwards in intervals of 9 to −21. Then, count backwards in intervals of 6 from −21 to −39. Use a number line if necessary.

Unit 1 All about integers

Important words

integers the number group that includes zero and all positive and negative numbers
**Introduction**

In Grade 7, we introduced integers by looking at opposites.

<table>
<thead>
<tr>
<th>Hot</th>
<th>Cold</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>Right</td>
</tr>
<tr>
<td>Fast</td>
<td>Slow</td>
</tr>
</tbody>
</table>

In Mathematics, every natural number on the number line has an opposite number that lies directly on the other side of 0. For example: The opposite of +3 is −3. We normally do not write +3, but only 3 when we refer to a positive integer. Do you notice that the distance from 0 to 3 is the same as the distance from 0 to −3 on the number line? Also notice that the number 0 is neither positive nor negative. The figure below illustrates what is meant by opposites in Mathematics!
Where do we use integers?

Negative numbers are part of our daily lives. In Grade 7, you learnt to interpret the temperatures given in weather forecasts. Let’s look at the thermometer: Can you see that $-10 \, ^\circ \text{C}$ and $10 \, ^\circ \text{C}$ lie on the opposite sides of $0 \, ^\circ \text{C}$. The same applies to $-20 \, ^\circ \text{C}$ and $20 \, ^\circ \text{C}$, $-30 \, ^\circ \text{C}$ and $30 \, ^\circ \text{C}$, and so on. We say these numbers are opposites of each other. We can also say that the one number is the negative of the other number. One of the coldest places in South Africa is Sutherland in the Northern Cape. Sutherland’s coldest day was on 12 July 2003, when the temperature dropped to $-16 \, ^\circ \text{C}$. This means it was 16 degrees Celsius below freezing point! Let’s compare this temperature to that on a nice summer day with a temperature of $28 \, ^\circ \text{C}$. We can do so using a number line:

Can you calculate the difference between these two temperatures? We will do more calculations like this in the next unit.

We also use negative numbers when we talk about altitude. For example, the highest place in the world is Mount Everest, which reaches $8 \, 848 \, \text{m}$ above sea level. The lowest place in the world is the Dead Sea between Jordan and Israel. The Dead Sea has an altitude of $-399 \, \text{m}$. This means it lies $399 \, \text{m}$ below sea level.

Another example of opposites is having and owing money. For example, Pontsho buys a bicycle from his friend for $R300$. However, Pontsho only has $R100$. He pays his friend $R100$, but still owes him $R200$. We can write Pontsho’s money situation as: $100 - 300 = -200$. 

The number line

Let’s go back to the number line: When you move along the number line from left to right, the numbers become bigger:

\(-3; -2; -1; 0; 1; 2; 3; \ldots\)

Based on this list of numbers, we can say that 3 is greater than all the numbers to its left:

\(3 > 2; 3 > 1; 3 > 0; 3 > -1; 3 > -2; 3 > -3; \text{ and so on.}\)

When we move from right to left along the number line, the numbers get smaller:

\(-6; -7; -8; -9; -10; \ldots\)

Based on this list of numbers, we can say that \(-10\) is smaller than all the numbers to its right:

\(-10 < -9; -10 < -8; -10 < -7; -10 < -6; \text{ and so on.}\)

The numbers on the number line are called integers.

Where do the integers fit?

In the first topic, we explained how each one of you belongs to groups of people who are related in some way. You are a member of your family, whom you live with, but your family is a part of the community in your area. In the same way, your community lives in a province.

You have already learnt about natural numbers (\(\mathbb{N}\)) and whole numbers (\(\mathbb{N}_0\)). The group of whole numbers contains all the natural numbers and the number 0. Now, all the whole numbers belong within a bigger group, called integers (\(\mathbb{Z}\)). Integers consist of all the negative numbers, 0 and all the positive numbers.

We can also write the numbers as a list:

\(\mathbb{N} = \{1; 2; 3; 4; \ldots\}\)

\(\mathbb{N}_0 = \{0; 1; 2; 3; 4; \ldots\}\)

\(\mathbb{Z} = \{-3; -2; -1; 0; 1; 2; 3; \ldots\}\)
Counting, ordering and comparing integers

Example
Use the number line to count forwards in intervals of 2 from \(-14\) to 6. Write out the sequence of numbers.

Solution
The sequence is: \(-14; -12; -10; -8; -6; -4; -2; 0; 2; 4; 6\)

Example
Complete the following chain diagram by counting backwards in intervals of 3 from 7 to \(-14\).

Solution
6 is greater than \(-28\) because 6 lies to the right of \(-28\).

Example
Compare the two numbers \(-9\) and \(-1\). Fill in \(<, >\) or \(=\) to make the following number sentence true: \(-9 \square -1\).

Solution
First, we plot the points on a number line.
Because $-9$ lies to the left of $-1$, we say that $-9$ is smaller than $-1$.
Therefore: $-9 < -1$.
Remember $-9 ^\circ C$ is colder (smaller) than $-1 ^\circ C$.

**Exercise 1**

1. Complete the following number grid:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$-144$</td>
<td>$-132$</td>
<td>$-84$</td>
</tr>
<tr>
<td>$-48$</td>
<td>$-24$</td>
<td>$0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$36$</td>
</tr>
</tbody>
</table>

2. Fill in the missing values on the following number line:

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$-64$</td>
<td>$-56$</td>
<td>$A$</td>
<td>$B$</td>
<td>$C$</td>
<td>$-24$</td>
<td>$D$</td>
</tr>
<tr>
<td>$E$</td>
<td>$F$</td>
<td>$G$</td>
<td>$24$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Write down the larger integer in each pair:
   a) $-4; -1$
   b) $0; -17$
   c) $3; -6$

4. a) Arrange the following numbers in ascending order (small to large):
   $-2; -64; 0; -14; -4; -16; -8$
   b) Arrange the following numbers in descending order (large to small):
   $3; 101; -101; -5; -1; -21; -20$

5. Compare the two numbers above the number line in each case. Write down the larger number:
   a)
   b)

6. You need to choose between the following amounts. Which would you prefer in each case?
   a) $-R200$ or $-R20$
   b) $-25c$ or $-250c$

7. Write down all the integers that are:
   a) greater than $-5$ but smaller than $2$
   b) less than $-5$ but greater than $-12$
   c) less than $4$ but greater than $-4$
   d) greater than $-25$ but less than $-18$.

8. Fill in the missing numbers in the following sequences:
   a) $-11; -7; ___; ___; 5; ___$
   b) $42; 34; ___; ___; 10; ___; ___; ___; -22$
   c) $18; 14; ___; ___; 2; ___; ___; -10; ___$

9. Fill in $<$, $>$ or $=$ between each pair of numbers:
   a) $-5 \square -9$
   b) $4 \square -1$
   c) $-3 \square -1$
   d) $-6\ 073 \square -4\ 627$
   e) $-89 \square -4$
   f) $-17 \square -71$

Plot the numbers on a number line, if necessary.
**Unit 2  Calculations using integers**

**Adding integers**

Let’s revise some addition calculations:

1. Add two positive numbers: $23 + 31 = 54$
2. Add two negative numbers: $-12 + (-23) = -35$
3. Add a negative and positive number: $-20 + 14 = -6$ and $18 + (-23) = -5$

The following examples explain calculations involving negative integers.

**Example**

Adding two or more negative numbers

Calculate the following:

1. $-6 + (-3)$
2. $1 + (-7) + (-4)$

**Solution**

1. If you regard negative numbers as debt, this means you owe one friend R6 and another friend R3. In total, you owe your friends R9. Can you see that when you add debt, it will just be more debt? Therefore, a negative number plus a negative number is a bigger negative number.

   We can show this on a number line. Remember **plus** means move to the right and **minus** means move to the left:

   ![Number Line Example 1](image1)

   So we say: $-6 + (-3) = -9$

2. In this case, we add three amounts of debt: you owe one friend R1, another friend R7 and a third friend R4. In total, you owe them R12.

   ![Number Line Example 2](image2)

   On a number line, it looks like this:

   So we say: $-1 + (-7) + (-4) = -12$

**Example**

Calculate the following:

1. $8 + (-2)$
2. $(-12) + 5$
3. $2 + 1 - 8 - 12 + 4 - 5 - 3 + 7$
Solution

1. If there is no sign in front of a number it means it is positive. So 8 is positive and means you have R8. However, you have R2 debt, which is money you still owe your friend. If you pay your friend the R2 you owe him, you will still have R6. Since this is money you have, your answer will be positive. On a number line, it looks like this:

So we say: \( 8 + (-2) = 6 \)

Can you see it is the same as subtracting two positive numbers? \( 8 - 2 = 6 \)

2. Here, you owe your friend R12, but you have R5. If you give him the R5 you have, you still owe him R7. Because you still have debt, your answer will be a negative number. On a number line, it looks like this:

So we say: \( -12 + 5 = -7 \)

3. When you need to add more than two numbers, first add all the positive numbers, then add all the negative numbers. Then, add the two answers. Let’s try ...

\[
2 + 1 - 8 - 12 + 4 - 5 - 3 + 7 \\
= 2 + 1 + 4 + 7 + (-8) + (-12) + (-5) + (-3) \\
= 14 + (-28) \\
= -14
\]
Subtracting integers

You have learnt that the additive inverse of +3 is −3, of 45 is −45, of −210 is +210, and so on. Remember that we normally write positive numbers without the ‘+’ symbol. So we write +210 as 210. When you add two additive inverses, for example: +3 + (−3), the answer is always 0. Now study the following:

\[21 − 16 = 5 \quad \text{(Read this as 21 minus 16 is equal to 5.)}\]
\[21 + (−16) = 5 \quad \text{(Read this as 21 plus negative 16 is equal to 5.)}\]

Now compare the two problems.

\[21 − 16\] and \[21 + (−16)\]: Can you see that subtracting 16 from 21 is the same as adding the additive inverse of 16 to 21?

Subtracting an integer from another integer is the same as adding its additive inverse to the original number. That is exactly what we are going to do in the next examples.

**Example**

Calculate:

1. \[3 − 6\]  
2. \[−2 − 7\]  
3. \[−15 − (−6)\]

**Solution**

1. \[3 − 6\] means: \[3 + (−6) = −3\] (Add the additive inverse to the number.)

2. \[−2 − 7\] means: \[−2 + (−7) = −9\] (Add the additive inverse to the number.)
3. \(-15 - (-6)\) means: \(-15 + (+6) = -9\) (Add the additive inverse to the number.)

Think of it like this: both negative numbers \((-15\) and \(-6\) mean debt. Subtracting means you are taking something away (making it less). So in this case, you are taking \(R6\) of debt away from \(R15\) of debt, leaving you with \(R9\) of debt.

Exercise 2

1. Calculate the following:
   a) \((-2) + (-4)\)
   b) \((-1) + (-8)\)
   c) \((-2) + (+4)\)
   d) \((-1) + (+8)\)
   e) \((+1) + (-8)\)
   f) \((-23) + (-17)\)

2. Complete the following flow diagrams:
   a) \(-9 \rightarrow \text{rule} \rightarrow 2 \rightarrow \text{output} \rightarrow 7\)
   b) \(-1 \rightarrow \text{rule} \rightarrow +(-13) + 7 \rightarrow \text{output} \rightarrow -12\)

3. Complete the following tables:
   a) \[
   \begin{array}{|c|c|c|c|c|c|c|c|}
   \hline
   x & 5 & 3 & 1 & -1 & -3 & -5 & -7 & -9 & -11 \\
   \hline
   x + (-12) & & & & & & & & & \\
   \hline
   \end{array}
   \]
   b) \[
   \begin{array}{|c|c|c|c|c|c|c|c|}
   \hline
   x & -13 & -22 & -29 & -31 & -45 & -49 & -55 & -60 \\
   \hline
   (-11) + x & & & & & & & & & \\
   \hline
   \end{array}
   \]

4. Calculate the following. First add all the positive and all the negative numbers together:
   a) \(12 - 8 - 5 - 1 - 4 - 2\)
   b) \(7 - 9 - 10 - 5 + 4 + 13\)
   c) \(-6 + 5 - 4 - 3 + 2 + 12 - 15\)
   d) \(-200 + 198 - 24 + 50\)

5. Write down the additive inverse of each of the following:
   a) \(8\)
   b) \(2\)
   c) \(-4\)
   d) \(-17\)
   e) \(-452\)
   f) \(1000\)
   g) \(-56\)
   h) \(32\)
   i) \(-1542\)

6. Find the sum of the numbers and their additive inverses.
   a) \(3 + (-3) = \)______
   b) \(-7 + (+7) = \)______
   c) \(5 + (-5) = \)______
   d) \(-780 + (+780) = \)______
   e) \(12 - 12 = \)______
   f) \(-28 + 28 = \)______

7. Calculate the following:
   a) \(4 - (+3)\)
   b) \(4 - (-3)\)
   c) \(-4 - (+3)\)
   d) \((-4) - (-3)\)
   e) \(7 + (+9)\)
   f) \(7 - (-9)\)
g) \(-7 - (+9)\)  \quad h) \(-7 - (-9)\)  \quad i) \((-2) - (-6)\)

j) \(+11 - (-6)\)  \quad k) \(5 - 8\)  \quad l) \(-12 - (15)\)

8. a) Subtract 16 from \(-8\).  
b) Subtract \(-11\) from \(-18\).

c) Subtract \(-5\) from 21.  
d) Subtract 23 from \(-13\).

9. Complete the following flow diagrams:

a) ![Flow diagram a]

b) ![Flow diagram b]

10. Complete the following tables:

a) \[\begin{array}{cccccccc}
   x & 29 & 18 & 16 & -1 & -4 & -8 & -9 & -12 & -20 \\
   x - 17 & & & & & & & & & \\
\end{array}\]

b) \[\begin{array}{cccccccc}
   (-4) - x & & & & & & & & & \\
\end{array}\]

11. Calculate the following:

a) \(-3 + 8 - 1 - 7 + 12 + 1\)

b) \(-15 + 20 - 14 - 13 + 15 + 4\)

c) \(-15 + 8 - 18 - 10 + 13 + 18\)

d) \(-12 + 2 - 13 - 14 + 18 + 10\)

e) \(-4 + 2 - 13 - 20 + 9 + 19\)

**Multiplying and dividing integers**

Multiplying positive integers works in the same way as multiplying whole numbers. For example: \((+4) \times (+3) = 4 \times 3 = 12\). But what happens when a positive number is multiplied by a negative number, or when we multiply two negative numbers? Let’s study some examples.

**Example**

Remember that multiplication is repeated addition. 
So \(2 \times 6 = 6 + 6 = 12\). When multiplying a positive number by a negative number, the same rule applies:

1. \(2 \times -6 = (-6) + (-6) = -12\)
2. \(5 \times -3 = (-3) + (-3) + (-3) + (-3) + (-3) = -15\)
3. \((-4 \times 7) = (-4) + (-4) + (-4) + (-4) + (-4) + (-4) + (-4) = -28\)
Example
Consider the pattern in the following sets of multiplication sums.

\[
\begin{align*}
3 \times 3 &= 9 \\
3 \times 2 &= 6 \\
3 \times 1 &= 3 \\
3 \times 0 &= 0 \\
3 \times -1 &= -3 \\
3 \times -2 &= -6 \\
3 \times -3 &= -9
\end{align*}
\]

Now, what happens when we multiply two negative numbers? For example, how do we work out \((-4) \times (-4)\)? We can explain this by finding patterns when multiplying numbers.

Example
Study the following multiplication table. Try to find a pattern. Then, complete the table on your own.

\[
\begin{array}{c|ccccccc}
\times & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\hline
-3 & & & & & & & \\
-2 & & & & & & & \\
-1 & & & & & & & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
2 & -6 & -4 & -2 & 0 & 2 & 4 & 6 \\
3 & -9 & -6 & -3 & 0 & 3 & 6 & 9
\end{array}
\]

Now compare your table to the following table.

\[
\begin{array}{c|ccccccc}
\times & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
\hline
-3 & 9 & 6 & 3 & 0 & -3 & -6 & -9 \\
-2 & 6 & 4 & 2 & 0 & -2 & -4 & -6 \\
-1 & 3 & 2 & 1 & 0 & -1 & -2 & -3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -3 & -2 & -1 & 0 & 1 & 2 & 3 \\
2 & -6 & -4 & -2 & 0 & 2 & 4 & 6 \\
3 & -9 & -6 & -3 & 0 & 3 & 6 & 9
\end{array}
\]

The bold numbers show a pattern on either side of 0. We can use this same pattern to complete the blocks that correspond with the numbers in blue. So we have:

\[
\begin{align*}
(-3) \times (-3) &= 9 \\
(-2) \times (-2) &= 4 \\
(-1) \times (-1) &= 1
\end{align*}
\]
From this exercise you must have noticed that when a negative integer is multiplied by a negative integer, the answer is a positive integer.

So, to solve the initial problem: 

\((-4) \times (-4) = 16.\)

Example

Another pattern that shows what happens when we multiply a negative integer by a negative integer is as follows:

\(-1 \times 3 = -3\)
\(-1 \times 2 = -2\)
\(-1 \times 1 = -1\)
\(-1 \times 0 = 0\)
\(-1 \times -1 = 1\)
\(-1 \times -2 = 2\)
\(-1 \times -3 = 3\)

To summarise

The rules are:

• a positive integer \(\times\) a positive integer = a positive integer
• a negative integer \(\times\) a negative integer = a positive integer
• a positive integer \(\times\) a negative integer = a negative integer
• a negative integer \(\times\) a positive integer = a negative integer.

Division is the inverse of multiplication. For example: If \(7 \times 9 = 63\), then \(63 \div 9 = 7\). The same applies for negative numbers, for example:

• if \(-7 \times 9 = -63\), then \(-63 \div 9 = -7\)
• if \(7 \times -9 = -63\), then \(-63 \div -9 = 7\)
• if \(-7 \times -9 = 63\), then \(63 \div -9 = -7\).

Can you see the pattern for the sign of the answer when dividing integers? Yes, they are the rules we used when multiplying:

• Dividing a positive and negative integer equals a negative integer.
• Dividing two negative integers equals a positive number.
Squares, cubes, square roots and cube roots

Squares and square roots
Squares and square roots for positive integers work the same way as they do for whole numbers. For example: \(5^2 = 5 \times 5 = 25\), so \(\sqrt{25} = 5\), because \(5 \times 5 = 25\).
Or \(8^2 = 8 \times 8 = 64\), so \(\sqrt{64} = 8\).

Now consider the following: \((-8)^2 = (-8) \times (-8) = 64\). So we see that \(\sqrt{64}\) can be \(+8\) or \(-8\). We write this as follows: \(\pm \sqrt{64} = \pm 8\)

Now, what about \(\sqrt{-64}\)? Could it be \(-8\)? Let’s check: \(-8 \times -8 = 64\) and not \(-64\). So the answer is not \(-8\). Could it be \(8\)? Let’s check: \(8 \times 8 = 64\) and not \(-64\). So the answer is not \(8\). It turns out that there is no integer that can solve this problem. This is true for the square root of all negative numbers. We say that the square root of a negative number is undefined.

You also need to be careful of the following:
\((-12)^2 = (-12) \times (-12) = 144,\)
but \(-12^2 = -(12 \times 12) = -144!\)
Can you see in the second calculation it is only 12 that is raised to the second power and not the negative sign?

You learnt in Grade 7 that the cube and cube root of a number are opposite operations. For example: If \(4^3 = 4 \times 4 \times 4 = 64\), then \(\sqrt[3]{64} = 4\), because \(4 \times 4 \times 4 = 64\). The same applies to integers.

**Example**
If \(5^3 = 5 \times 5 \times 5 = 125\), then \(\sqrt[3]{125} = 5\).
If \((-5)^3 = -5 \times -5 \times -5 = -125\), then \(\sqrt[3]{-125} = -5\).

Do you notice the difference between the square and cube root? We are not able to find the square root of a negative number. However, the cube root of a negative number is always negative.
Exercise 3

1. Complete the following:

<table>
<thead>
<tr>
<th>Factors</th>
<th>Product</th>
<th>Factors</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+2) × (+4)</td>
<td>=</td>
<td>+8</td>
<td>(-2) × (+4)</td>
</tr>
<tr>
<td>(+2) × (+3)</td>
<td>=</td>
<td>+?</td>
<td>(-2) × (+3)</td>
</tr>
<tr>
<td>(+2) × (+2)</td>
<td>=</td>
<td>+?</td>
<td>(-2) × (+2)</td>
</tr>
<tr>
<td>(+2) × (+1)</td>
<td>=</td>
<td>+?</td>
<td>(-2) × (+1)</td>
</tr>
<tr>
<td>(+2) × 0</td>
<td>=</td>
<td>0</td>
<td>(-2) × 0</td>
</tr>
<tr>
<td>(+2) × (−1)</td>
<td>=</td>
<td>−2</td>
<td>(-2) × (−1)</td>
</tr>
<tr>
<td>(+2) × (−2)</td>
<td>=</td>
<td>−?</td>
<td>(-2) × (−2)</td>
</tr>
<tr>
<td>(+2) × (−3)</td>
<td>=</td>
<td>−?</td>
<td>(-2) × (−3)</td>
</tr>
<tr>
<td>(+2) × (−4)</td>
<td>=</td>
<td>−?</td>
<td>(-2) × (−4)</td>
</tr>
</tbody>
</table>

2. Calculate the following:

a) (5)(−3) b) (−5)(−3) c) −5 × +3
d) (−4)(−8) e) (6)(−9) f) (−4)(5)
g) 3 × 2 × 5 × (−1) h) (−6)(−4)(+2) i) (−2)(−1)(−3)(−3)
j) −3 × 2 × −1 × 4 k) 72 ÷ (−8) l) (−63) ÷ (−9)
m) (−9) ÷ 9 n) (−60) ÷ 0 o) 48 ÷ (−6)
p) (−1) ÷ (−1) q) (−5) ÷ 5 r) 0 ÷ (−7)
s) $\frac{64}{−16}$ t) $\frac{−100}{−25}$ u) $−\frac{24}{8}$

3. Calculate the following:

a) Subtract 25 from 6. b) (−2)(−10)(3) ÷ (−3)(4)
c) From −8 subtract 18. d) 16 − 8 × −4 + 2 ÷ 2
e) Decrease 12 by 16. f) 3(4 − 10) ÷ (2 − 4) − 4
g) Increase −11 by 19. h) 6 + 21 ÷ 7 − 7 × (−6)(0) + 9
i) −4(4 − 1)(−7 − 9) j) −2 − (−10) ÷ (−5)
k) 7 − (3)(2) + 9(−2) l) 0 − 22 + −32 − 42 + (−4)(2)
m) Find the product of (−17 + 4) and (−16 + 15)(2).

n) Find the sum of 7 and −10. Then, subtract 2(−3)(2).
o) Find the difference between 2(−3)(2) and (3)(3).

4. Calculate the following:

a) (−4)$^2$ b) (−12)$^2$ c) (−15)$^2$
d) (−1)$^3$ e) (−3)$^3$ f) (−6)$^3$
g) −8$^2$ h) −11$^2$ i) −4$^3$

5. Calculate the following roots:

a) $\sqrt{81}$ b) $\sqrt{(−81)}$
c) $\sqrt[3]{27}$ d) $\sqrt[3]{(−27)}$
e) $\sqrt[3]{(−125)}$ f) $\sqrt[3]{(−1000)}$
**Unit 3  Properties of integers**

**Introduction**
First, we will revise what you have already learnt about the properties of whole numbers and integers. Then, we explore the properties of integers in more detail.

**The commutative property**
The commutative property tells us that the order does not matter when we add or multiply numbers. In both cases, the answer remains the same.

**Example**
**Whole numbers:**
\[23 + 7 = 30 \text{ and } 7 + 23 = 30\]
\[45 \times 2 = 90 \text{ and } 2 \times 45 = 90\]

**Integers:**
\[(-5) + (-6) = -11 \text{ and } (-6) + (-5) = -11\]

**The associative property**
The associative property tells us that we can group numbers together in different ways without affecting the result. This property applies to addition and multiplication.

**Example**
**Whole numbers:**
\[(23 + 7) + 20 = 30 + 20 = 50 \text{ and } 23 + (7 + 20) = 23 + 27 = 50\]
\[(7 \times 5) \times 2 = 35 \times 2 = 70 \text{ and } 7 \times (5 \times 2) = 7 \times 10 = 70\]

**Integers:**
\[(-4 + 8) + (-7) = 4 + (-7) = -3 \text{ and } -4 + [8 + (-7)] = -4 + 1 = -3\]

**The distributive property**
The distributive property allows us to redistribute numbers and still obtain the same answer.

**Example**
**Whole numbers:**
\[5 \times (30 + 2) = (5 \times 30) + (5 \times 2) = 150 + 10 = 160\]
\[4 \times (50 - 2) = (4 \times 50) - (4 \times 2) = 200 - 8 = 192\]
The properties of the number 0 and 1

- We call 0 the identity element for addition because when we add or subtract 0 from a number, the number remains the same.
  \[39 + 0 = 39; \quad 76 - 0 = 76; \quad -10 + 0 = -10; \quad -9 - 0 = -9\]
- \[64 \times 0 = 0\]
- \[\frac{0}{12} = 0\]
- \[\frac{12}{0}\] has no answer. We say that dividing by 0 is undefined.
- We call 1 the identity element for multiplication, because when we multiply a number by 1, the number always stays the same.
  \[55 \times 1 = 55\]

More about the properties of integers

Now let’s explore the properties of integers in more detail.

**Example**

Calculate:

a) \[8 \times (-6)\] and \((-6) \times 8\]

b) \[(-9) \times (-12)\] and \((-12) \times (-9)\]

c) Is there a pattern in the first two questions?

**Solution**

a) \[8 \times (-6) = -48\] and \((-6) \times 8 = -48\]

b) \[(-9) \times (-12) = 108\] and \((-12) \times (-9) = 108\]

c) When we multiply two integers, the order of the numbers is not important.

**Example**

Calculate:

a) \[(-11 \times 2) \times -4\] and \[-11 \times [2 \times (-4)]\]

b) \[-25 \times (-4 \times -2)\] and \[(-25 \times -4) \times -2\]

c) Is there a pattern in the first two questions?

**Solution**

a) \[(-11 \times 2) \times -4 = -22 \times -4 = 88\] and \[-11 \times [2 \times (-4)] = -11 \times -8 = 88\]

b) \[-25 \times (-4 \times -2) = -25 \times 8 = -200\] and \[(-25 \times -4) \times -2 = 100 \times -2 = -200\]

c) When multiplying more than two numbers, the grouping of the numbers is not important.
Example
Calculate the following by:
  a) first doing the part in brackets
  b) using the distributive property.

1. \(-3 \times (4 + 12)\)
2. \(8 \times [(-9) + (-7)]\)
3. \(-3[12 - (-9)]\)
4. \(-7(-2 - 10)\)
5. What did you notice about your answers to a) and b) in each case?

Solution
1. a) \(-3 \times (4 + 12) = -3 \times 16 = -48\)
   b) \(-3 \times (4 + 12) = (-3 \times 4) + (-3 \times 12) = -12 + (-36) = -48\)
2. a) \(8 \times [(-9) + (-7)] = 8(-16) = -128\)
   b) \(8 \times [(-9) + (-7)] = (8 \times -9) + (8 \times -7) = -72 + (-56) = -128\)
3. a) \(-3[12 - (-9)] = -3(12 + 9) = -3 \times 21 = -63\)
   b) \(-3[12 - (-9)] = (-3 \times 12) - (-3 \times -9) = -36 - (27) = -63\)
4. a) \(-7(-2 - 10) = -7(-12) = 84\)
   b) \(-7(-2 - 10) = (-7 \times -2) - (-7 \times 10) = 14 - (-70) = 14 + 70 = 84\)
5. When working with integers, you may redistribute the numbers. The answer remains the same. This implies that the distributive property also applies to integers.

The answers were the same every time!

Example
Calculate:
1. \(-6 \times 0\)
2. \(0 \div 12\)
3. \(\frac{0}{-12}\)
4. \(\frac{5}{0}\)
5. \(\frac{-5}{0}\)
6. \(-6 \times 1\)

Solution
1. 0
2. 0
3. 0
4. Undefined
5. Undefined
6. -6

So we can see that the properties of 0 and 1 also apply to integers.
Inverse operations

Inverse operations are a pair of operations that reverse each other. You use inverse operations to check whether a calculation is correct.

- Addition and subtraction are inverse operations.
  For example, if $460 + 340 = 800$, the two possible inverse operations are $800 - 340 = 460$ or $800 - 460 = 340$.

- Multiplication and division are inverse operations.
  For example, if $150 \times 40 = 6000$, the two possible inverse operations are $6000 \div 40 = 150$ or $6000 \div 150 = 40$.

Addition and subtraction are inverse operations and this applies not only to positive, but also to negative numbers, in other words to the integers.

Example

Calculate the following. Then check your answers using an inverse operation.

1. $-5 + (-9)$
2. $34 - 14$
3. $15 - (-7)$

Solution

1. $-5 + (-9) = -14$
   
   Check: $-14 - (-9) = -14 + (+9) = -5$

2. $34 - 14 = 20$
   
   Check: $20 + 14 = 34$

3. $15 - (-7) = 15 + 7 = 22$
   
   Check: $22 + (-7) = 15$
Multiplication and division as inverse operations

Study the following examples.

**Example**
Calculate the following and then check your answer using an inverse operation:

1. \(125 \times 8\)
2. \(400 \div 8\)
3. \((-12) \times 25\)
4. \((-100) \div (-5)\)

**Solution**

1. \(125 \times 8 = 1000\)
   - Check: \(1000 \div 8 = 125\)

2. \(400 \div 8 = 50\)
   - Check: \(50 \times 8 = 400\)

3. \((-12) \times 25 = -300\)
   - Check: \((-300) \div 25 = -12\)

4. \((-100) \div (-5) = 20\)
   - Check: \(20 \times (-5) = -100\)

**Exercise 4**

1. Complete the following number sentences. The first one has been completed for you.
   
   a) \(-1\,600 + 400 = -1\,200\) and \(400 + (-1\,600) = _____\)
   
   b) \(-52 \times 21 = _____\) and \(21 \times -52 = _____\)
   
   c) \((-22 \times -3) \times 10 = _____\) and \(-22 \times (-3 \times 10) = _____\)
   
   d) \(-63 \times (11 \times _____) = 15\,939\) and \((-63 \times 11) \times -23 = _____\)
   
   e) \(2\,200 + _____ = 2\,000\) and \(-200 + _____ = 2\,000\)
   
   f) _____ + (-40 + 20) = -100 and \([-80 + (-40)] + 20 = _____\)
   
   g) \(13 \times -400 = _____\) and \(-400 \times 13 = _____\)

2. Use the distributive property to complete the following:
   
   a) \(5 \times [9 + (-2)] = (5 \times _____) + (5 \times _____) = _____ + _____ = _____\)
   
   b) \(50(-80 + 5) = (_____ \times -80) + (_____ \times 5) = _____ + _____ = _____\)
   
   c) \(-8 \times 38 = -8 \times (40 - ____)) = (-8 \times 40) - (-8 \times ____))
   
   = _____ - _____ = _____
   
   d) \(48 \times -9 = (50 - ____)) \times -9 = (50 \times -9) - (2 \times ____))
   
   = _____ - _____ = _____

3. Complete the following number sentences.
   
   a) \(-238 \times _____ = -238\)
   b) \(-649 + 0 = _____\)
   
   c) _____ + (-8\,300) = -8\,300\)
   d) \(1 \times _____ = -87\)
   
   e) \(0 \div (-96) = _____\)
   f) \(-900 \div 0 = _____\)

4. Check the solution to the following calculations using the inverse operation.
   
   a) \(-58 + 12 = -46\) Write down a subtraction sentence.
   
   b) \(84 \div (-16) = 100\) Write down an addition sentence.
   
   c) \(132 \div (-11) = -12\) Write down a multiplication sentence.
   
   d) \(-75 \times -4 = 300\) Write down two division sentences.
Unit 4  Solving problems

Where do we use integers?
The following table shows a few of the real-life situations in which we use integers.

<table>
<thead>
<tr>
<th>Example</th>
<th>Positive number</th>
<th>Negative number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>Above 0 °C</td>
<td>Below 0 °C</td>
</tr>
<tr>
<td>Altitude of places in the world</td>
<td>Above sea level</td>
<td>Below sea level</td>
</tr>
<tr>
<td>Business transaction</td>
<td>Profit</td>
<td>Loss</td>
</tr>
<tr>
<td>Budgets</td>
<td>Surplus</td>
<td>Debt</td>
</tr>
<tr>
<td>Bank statement</td>
<td>Credit</td>
<td>Debit</td>
</tr>
</tbody>
</table>

Companies can be private or public companies. A public company is listed on a stock exchange. As an investor, you can then buy and sell shares in that company.

Banks provide a safe place to keep money. You can open a bank account at the bank in which you can save your money. When you pay money into the account, you make a deposit. When you take money out of the account, you make a withdrawal. Every month, the bank sends you a statement showing the transactions on the account. A transaction occurs whenever money goes into or out of the account. The statement also shows the balance on the account. The balance is the amount of money in the account.

Exercise 5

1. Express each of the following as an integer:
   a) A debt of R420
   b) A profit of R84
   c) A temperature of 12 °C above zero
   d) 823 m below sea level
   e) A credit of R1 270
   f) A temperature of 5 °C below zero
   g) A surplus of R268
   h) 2 300 m above sea level
   i) A loss of R26
   j) A debit of R540
2. a) Two towns have minimum temperatures of 3 ºC and −13 ºC, respectively. Which town experiences the warmer temperature?
   
b) Divers A and B are diving at −20 m and −40 m below sea level, respectively. Which diver is diving at the deeper level?

   c) Suzanne has no money in her bank account. Mary has −R40 in her bank account. Who is in a better financial position?

3. The following table shows the minimum and maximum temperatures of a few towns and cities in South Africa. The temperature range is the difference between the maximum and minimum temperatures. Complete the table.

<table>
<thead>
<tr>
<th>Town</th>
<th>Minimum temperature (ºC)</th>
<th>Maximum temperature (ºC)</th>
<th>Temperature range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polokwane</td>
<td>5</td>
<td>26</td>
<td></td>
</tr>
<tr>
<td>Madadeni</td>
<td>−3</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Khayelitsha</td>
<td>27</td>
<td></td>
<td>15</td>
</tr>
<tr>
<td>Johannesburg</td>
<td>−2</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>Kimberley</td>
<td>14</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>Mmabatho</td>
<td>0</td>
<td></td>
<td>22</td>
</tr>
<tr>
<td>Mhluzi</td>
<td>−1</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>Mangaung</td>
<td>−7</td>
<td>9</td>
<td>24</td>
</tr>
<tr>
<td>Bhisho</td>
<td>25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sutherland</td>
<td>13</td>
<td>21</td>
<td></td>
</tr>
</tbody>
</table>

4. Kgomotso has saved R95. She buys a watch from her friend for R110.
   
a) How much does she still owe her friend?
   
b) Write down a mathematical sentence that shows Kgomotso’s financial position.

5. Use your calculator to calculate the following:
   Shaheeda owns 200 shares in the company Gold Bullion, 150 in the company Platinum Purposes, and 60 in the company Copper Corporation. The opening and closing prices for the shares on a certain day are given in the table on the following page. The opening price shows the share price at the start of the day. The closing price shows the share price at the end of the day.
<table>
<thead>
<tr>
<th>Share</th>
<th>Opening price</th>
<th>Closing price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold Bullion</td>
<td>276</td>
<td>288</td>
</tr>
<tr>
<td>Platinum Purposes</td>
<td>297</td>
<td>252</td>
</tr>
<tr>
<td>Copper Corporation</td>
<td>183</td>
<td>159</td>
</tr>
</tbody>
</table>

Write down the change in the value of her shares for the day. If the share value has increased, show this as a positive change. If the share value has decreased, show this as a negative change.

6. Below is Mr Makofane’s bank account statement. Calculate his balance. Explain his financial position. If you were his financial advisor, what advice would you give him?

<table>
<thead>
<tr>
<th>Get-u-Rich Credit Card Account</th>
<th>Date</th>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mr T. Makofane</td>
<td>1/10</td>
<td>Balance brought forward</td>
<td>345,87–</td>
</tr>
<tr>
<td></td>
<td>2/10</td>
<td>Deposit Brooklyn ATM</td>
<td>1 500,00</td>
</tr>
<tr>
<td></td>
<td>5/10</td>
<td>Pick n Pay</td>
<td>63,79–</td>
</tr>
<tr>
<td></td>
<td>8/10</td>
<td>Cash Withdrawal Menlyn ATM</td>
<td>250,00–</td>
</tr>
<tr>
<td></td>
<td>14/10</td>
<td>Hang Ten Clothing</td>
<td>99,99–</td>
</tr>
<tr>
<td></td>
<td>14/10</td>
<td>Mr Price Clothes</td>
<td>139,98–</td>
</tr>
<tr>
<td></td>
<td>21/10</td>
<td>Cash Withdrawal Menlyn ATM</td>
<td>250,00–</td>
</tr>
<tr>
<td></td>
<td>22/10</td>
<td>Spar Groceries</td>
<td>255,69–</td>
</tr>
<tr>
<td></td>
<td>24/10</td>
<td>Deposit Internet Acc 422 3257 6831</td>
<td>150,00</td>
</tr>
<tr>
<td></td>
<td>25/10</td>
<td>MTN Direct</td>
<td>129,00–</td>
</tr>
<tr>
<td></td>
<td>27/10</td>
<td>Cash Withdrawal Menlyn ATM</td>
<td>150,00–</td>
</tr>
<tr>
<td></td>
<td>29/10</td>
<td>Sport Shoerama</td>
<td>299,00–</td>
</tr>
<tr>
<td></td>
<td>31/10</td>
<td>Pick n Pay</td>
<td>87,45–</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Balance:</td>
<td></td>
</tr>
</tbody>
</table>
**Topic 3 Exponents**

**In this topic you will learn to:**
- revise squares and their square roots and cubes and their cube roots
- compare and represent whole numbers in exponential form
- compare and represent integers in exponential form
- compare and represent numbers in scientific notation
- establish general laws of exponents
- recognise and use laws of operations
- perform calculations involving all four operations
- calculate the squares, cubes, square roots and cube roots of rational numbers
- solve problems in contexts involving numbers in exponential form.

**What you already know**

1. Write the following in exponential form (do not calculate the answer):
   - a) \(7 \times 7 \times 7\)
   - b) \(12 \times 12 \times 12 \times 12 \times 12\)
   - c) \(p \times p \times p \times p\)
   - d) \(a \times a \times a \times a \times a \times a \times a\)

2. Write the following in expanded form:
   - a) \(5^3\)
   - b) \(n^5\)
   - c) \(y^6\)

**Unit 1 Compare and represent numbers in exponential form**

**Introduction**

Multiplication is a short way to represent repeated addition. For example, we can write \(7 + 7 + 7 + 7 + 7 = 42\) in short as \(6 \times 7 = 42\). The exponential form is the short way for writing repeated multiplication.

For example, \(7 \times 7 \times 7 \times 7 \times 7 = 7^6\).

Study the following pattern:
- \(10 = 10^1\)
- \(100 = 10 \times 10 = 10^2\)
- \(1000 = 10 \times 10 \times 10 = 10^3\)
- \(10\,000 = 10 \times 10 \times 10 \times 10 = 10^4\)
- \(100\,000 = 10 \times 10 \times 10 \times 10 \times 10 = 10^5\)
- \(1\,000\,000 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6\)
Let’s consider $10^6$.

We read $10^6$ as ‘10 to the power of 6’ or ‘10 raised to the power of 6’.

The exponent of 6 indicates the number of times the factor 10 is multiplied by itself. Remember that, instead of writing $5^1$, we only write 5. So when a number does not have an exponent, it is because the exponent is actually one.

In general, we say $a^b$ is $a$ to the power of $b$. Here, $a$ is the base and $b$ is the exponent. Once again, $b$ indicates the number of times the factors are multiplied by themselves.

For example: $a^3 = a \times a \times a$ and $a^5 = a \times a \times a \times a \times a$.

### Squares and square roots

Do you remember the square numbers? To find the square numbers, we multiply a number by itself.

**Example:**

Find the first six square numbers.

**Solution**

The first six square numbers are: $1^2; 2^2; 3^2; 4^2; 5^2; 6^2$

In simplified form, the sequence is: $1; 4; 9; 16; 25; 36$

The figure alongside illustrates the geometrical meaning of the square number 9. Here, 9 is the result of 3 squared ($3^2$). Notice that the figure is a square.

A square is a two-dimensional (2D) figure. The area of this square is:

$3$ units $\times$ $3$ units $= 9$ square units. To work this out on your calculator, key in: $3; x^2; =$.

Finding the square root of a number is the inverse operation of finding the square of a number. This relationship is shown in the figure alongside. To work out the square root of 9 on your calculator, key in: $\sqrt{9}; 9; =$.
Example
If the square of 3 is 9, then the square root of 9 is 3. We write this as:
if $3^2 = 9$ then $\sqrt{9} = 3$.

The two square root signs $\sqrt{}$ and $\sqrt[3]{\ }$ mean the same thing. However, we do not usually write the 2 on the outside of the root sign.

Cubes and cube roots
The cube of a number is that number multiplied by itself three times. For example, when calculating $2^3$, we have $2 \times 2 \times 2 = 8$.

Example
Find the cubes of 1, 2 and 3.

Solution
$1^3; 2^3; 3^3$
In simplified form, the sequence is: 1; 8; 27.

Let’s look more closely at $3^3 = 27$. We read this as ‘the cube of 3 is 27’ or ‘3 cubed is 27’ or ‘3 to the power of 3 is 27’. The figure alongside shows the geometrical meaning of 3 cubed. The figure is a three-dimensional (3D) object called a cube. The volume of this cube is $3 \times 3 \times 3 = 27$ cubed units. To work this out on your calculator, key in: $3; x^3; =$.

Finding the cube root of a number is the inverse of finding the cube of a number. The diagram on the left shows the relationship between a cube and the cube root of a number.

Example
1. Calculate $\sqrt[3]{64}$. Give a reason for your answer.
2. Check your answer using your calculator.

Solution
1. $\sqrt[3]{64} = 4$, because $4 \times 4 \times 4 = 64$.
2. Key in: $\sqrt[3]{\ }; 64; =$
   The answer is 4.
Example
The following table shows a few numbers in exponential form, expanded form and then the actual value. Note that you do not have to be able to calculate the value. Use your calculator to find the value. For example, to calculate $32^5$ on your calculator, key in: $32 \times 32 \times 32 \times 32 \times 32$.

<table>
<thead>
<tr>
<th>Exponential form</th>
<th>Expanded form</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$32^2$</td>
<td>$32 \times 32$</td>
<td>1 024</td>
</tr>
<tr>
<td>$32^3$</td>
<td>$32 \times 32 \times 32$</td>
<td>32 768</td>
</tr>
<tr>
<td>$32^4$</td>
<td>$32 \times 32 \times 32 \times 32$</td>
<td>1 048 576</td>
</tr>
<tr>
<td>$32^5$</td>
<td>$32 \times 32 \times 32 \times 32 \times 32$</td>
<td>33 554 432</td>
</tr>
</tbody>
</table>

Example
1. Represent $53 \times 53 \times 53 \times 53 \times 53 \times 53$ in exponential form. (Do not calculate the answer.)
2. Simplify: $b \times b \times b \times b \times b$
3. Write $66^4$ in expanded form. (Do not calculate the answer.)
4. Write $m^3$ in expanded form.

Solution
1. $53 \times 53 \times 53 \times 53 \times 53 \times 53 = 53^6$
2. $b \times b \times b \times b \times b = b^5$
3. $66^4 = 66 \times 66 \times 66 \times 66$
4. $m^3 = m \times m \times m$

These are some very important facts that you need to remember:
- $100^2 = 100$
- $12^2 = 12 \times 12 = 144$
  (and not $12 \times 2 = 24$)
- $1^3 = 1 \times 1 \times 1 = 1$
  (and not $1 \times 3 = 3$)
- $\sqrt{81} = 9$ because $9^2 = 81$.
- $\frac{1}{3}27 = 3$ because $3^3 = 27$.
- The square of 9 is 81, but the square root of 9 is 3.

Exercise 1
1. a) Write down all the square numbers from 1 to 144.
   b) Fill in the missing values: 1; 8; 27; ___; ____
   c) Write 9 to the fourth power in exponential form.
   d) Write $y$ cubed in exponential and expanded form.
2. Calculate the following square and cube numbers:
   a) $4^2$  
   b) $12^2$  
   c) $5^2$  
   d) $1^2$
   e) $10^2$  
   f) $13^2$  
   g) $3^3$  
   h) $1^3$
   i) $2^3$  
   j) $6^3$  
   k) $4^3$  
   l) $5^3$
3. Determine the following roots. Check your answers through multiplication.
   a) $\sqrt{25}$  
   b) $\sqrt{49}$  
   c) $\sqrt{1}$  
   d) $\sqrt{9}$  
   e) $\sqrt{121}$  
   f) $\sqrt{81}$  
   g) $\sqrt{169}$  
   h) $\sqrt{100}$
   i) $3\sqrt{1}$  
   j) $3\sqrt{8}$  
   k) $3\sqrt{64}$  
   l) $3\sqrt{27}$
   m) $3\sqrt{125}$  
   n) $3\sqrt{216}$
4. What is the meaning of the following powers? (Do not calculate the final answer.)
   a) $17^5$  
   b) $85^6$  
   c) $1006^4$  
   d) $20000^3$
5. Represent the following numbers in exponential form:
   a) $75 \times 75 \times 75 \times 75$
   b) $102 \times 102 \times 102 \times 102 \times 102 \times 102$
6. Calculate the following:
   a) $9^2$  
   b) $9 \times 2$  
   c) $9 \times 9$  
   d) $2^9$

Represent and compare integers in exponential form

Now that you have revised exponents using whole numbers, let’s investigate what happens when we work with negative numbers. We will start with an example so that we can find a pattern.

Example
1. $(-2)^1 = -2$
2. $(-2)^2 = (-2) \times (-2) = 4$
3. $(-2)^3 = (-2) \times (-2) \times (-2) = -8$
4. $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2) = 16$
5. $(-2)^5 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -32$
6. $(-2)^6 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = 64$
7. $(-2)^7 = (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) \times (-2) = -128$

Can you see the pattern? If not, here is a clue: look at the exponent and the sign of the answer. What did you find?
- If the exponent is an odd number (1, 3, 5, 7, …), the answer is negative.
- If the exponent is an even number (2, 4, 6, …), the answer is positive.

Example
Predict whether the following will have a positive or negative answer.
1. $(-15)^4$  
2. $(-15)^3$